Whole-Body Control for Robots in the Real World

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Inverse Kinematics: New Method for Minimum Jerk Trajectory Generation

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Inverse Kinematics Problem

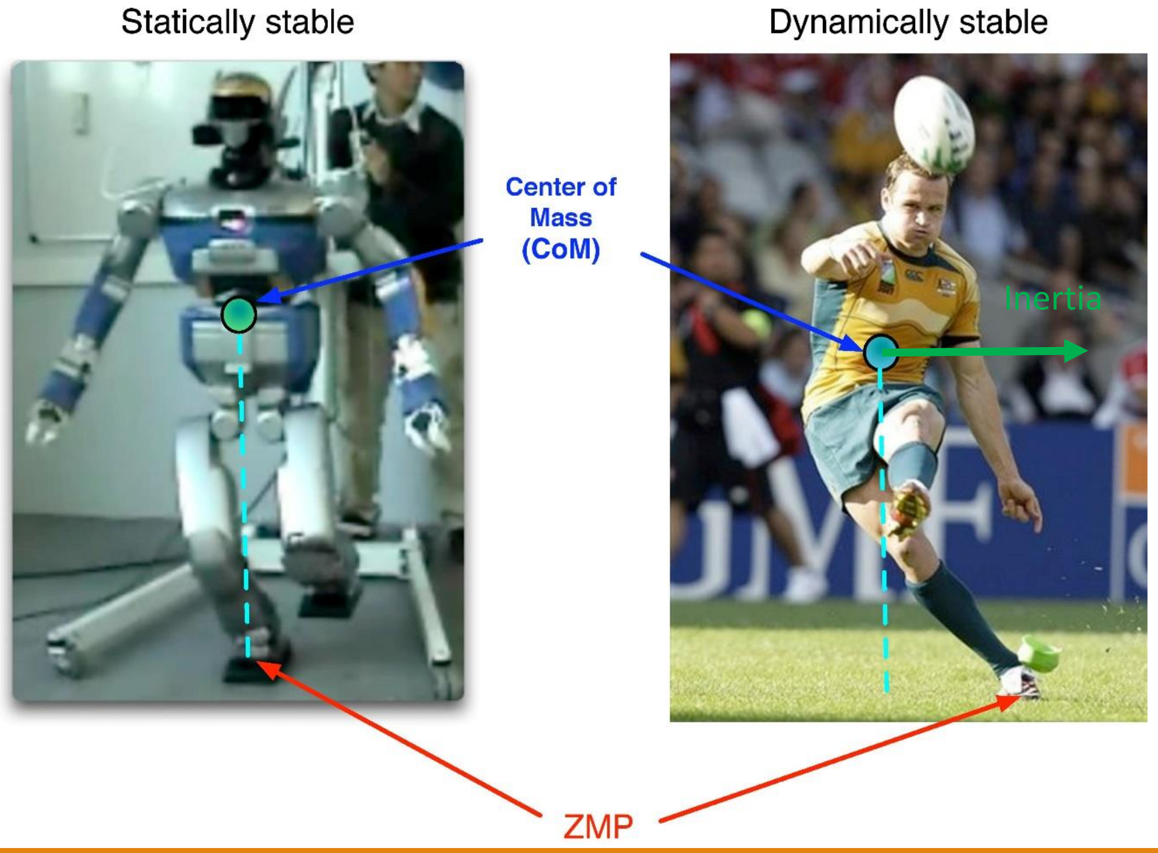
- Formulation and classical resolution
  - Original optimization problem : $\min_{\dot{q}} \dot{q}_t{}^T Q \dot{q}_t$ subject to $J\dot{q}_t = \dot{r}_t$

    ⟹ Minimize joints velocity.
  - Pseudo-inverse technique : $\dot{q}_t = J^+ \dot{r}_t$ with $J^+ = J^T (JJ^T)^{-1}$

    ⟹ **Joints velocity** as the control parameter.
- Additional constraints
  - Velocity/joints limits and obstacles inequality : $A\dot{q}_t \leq b$
  - Quadratic Programming (QP) solver.
- **No constraints** on the **acceleration** or the **jerk**

Zero Moment Point (ZMP)

- Humanoid robots dynamic stability control.



- ➢ Stability ↔ ZMP inside the polygon of support.
- ➢ ZMP depends on joint accelerations.

Discontinuous accelerations

↓

Discontinuous ZMP trajectory

↓

Imbalance

Joints jerk control

- Objective : To keep joints jerk within bounds for continuous acceleration.
- Relation between position, velocity, acceleration and jerk :

$$\begin{bmatrix} q_t \\ \dot{q}_t \\ \ddot{q}_t \end{bmatrix} = \begin{bmatrix} I_n & T\,I_n & \frac{T^2}{2}\,I_n \\ 0 & I_n & T\,I_n \\ 0 & 0 & I_n \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \dot{q}_{t-1} \\ \ddot{q}_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{T^3}{6} I_n \\ \frac{T^2}{2} I_n \\ T\,I_n \end{bmatrix} \dddot{q}_t$$

$= u_t$ (control)

$= X_t$ (Vector to be computed)

$= X_{t-1}$ (Current configuration)

$$\dot{q}_t = \dot{q}_{t-1} + T\,\ddot{q}_{t-1} + \frac{T^2}{2} u_t$$

Relationship between **velocity** and **jerk**

Problem reformulation

$$\min_{\dot{q}} \dot{q}_t^{\,T} Q \dot{q}_t$$
subject to
$$J\dot{q}_t = \dot{r}_t$$
$$A\dot{q}_t \leq b$$

$$\dot{q}_t = \dot{q}_{t-1} + T\,\ddot{q}_{t-1} + \frac{T^2}{2} u_t$$

$$\min_{u_t} u_t^T Q u_t$$
subject to
$$J u_t = \tilde{\dot{r}}_t$$
$$A\, u_t \leq \tilde{b}$$
$$u^- \leqslant u_t \leqslant u^+$$

Limits of the jerk vector

Where :

$$\tilde{\dot{r}}_t = \frac{2}{T^2}\left(\dot{r}_t - J\dot{q}_{t-1} - J\,T\,\ddot{q}_{t-1}\right)$$
$$\tilde{b} = \frac{2}{T^2}\left(b - A\dot{q}_{t-1} - A\,T\,\ddot{q}_{t-1}\right)$$

Implementation

Control algorithm

r_{t-T}, X_{t-T}

$$X_0 = \begin{bmatrix} q_0 \\ 0 \\ 0 \end{bmatrix}$$

$t = T$

$$J = f(r_{t-T}, q_{t-T})$$

J, X_{t-T}

$$\min_{u_t} u_t^T Q u_t$$

subject to

$$J u_t = \tilde{\ddot{r}}_t$$

$$A\, u_t \leq \tilde{b}$$

$$u^- \leqslant u_t \leqslant u^+$$

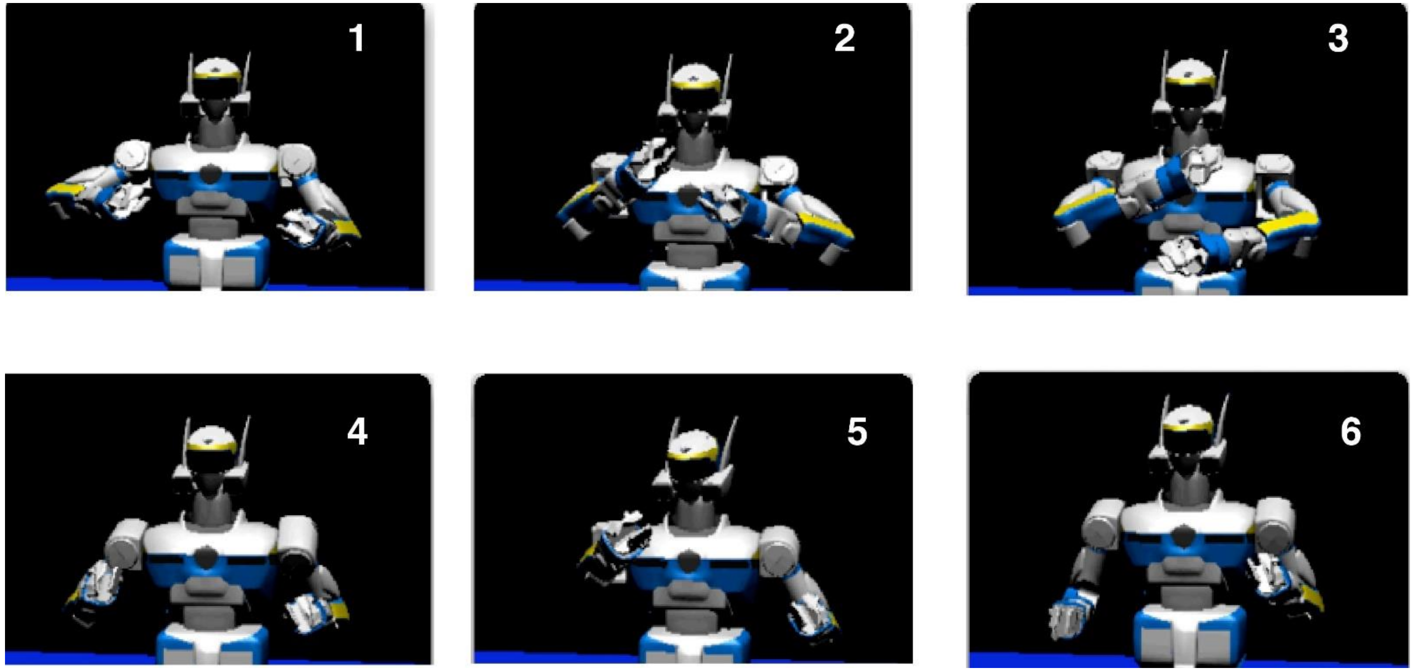
$\ddot{q}, X_{t-T}$

$$\begin{bmatrix} q_t \\ \dot{q}_t \\ \ddot{q}_t \end{bmatrix} = \begin{bmatrix} I_n & T\, I_n & \frac{T^2}{2}\, I_n \\ 0 & I_n & T\, I_n \\ 0 & 0 & I_n \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \dot{q}_{t-1} \\ \ddot{q}_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{T^3}{6} I_n \\ \frac{T^2}{2} I_n \\ T\, I_n \end{bmatrix} \dddot{q}_t$$

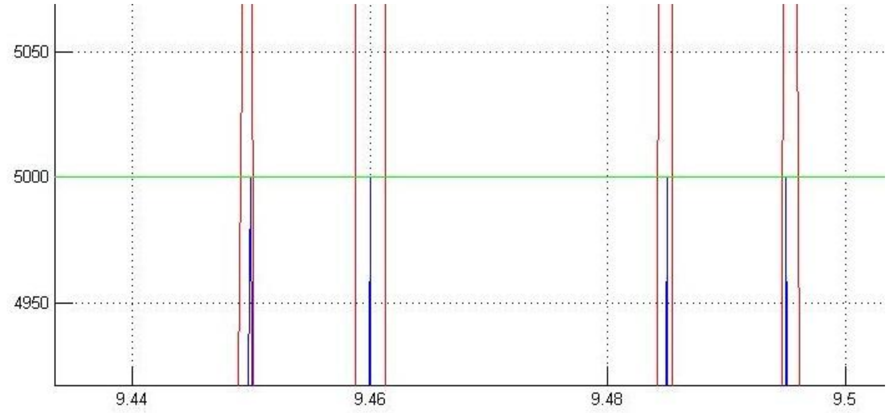
X_t

Experimental results

- Objectives:
  - Execute a boxing motion.
  - Follow end-effector Cartesian trajectory.
  - Respect limits on jerk (5000 rad s^{-3}).
- Tools:
  - Virtual robot HRP-2 using OpenHRP platform.
  - Data treatment on Matlab.

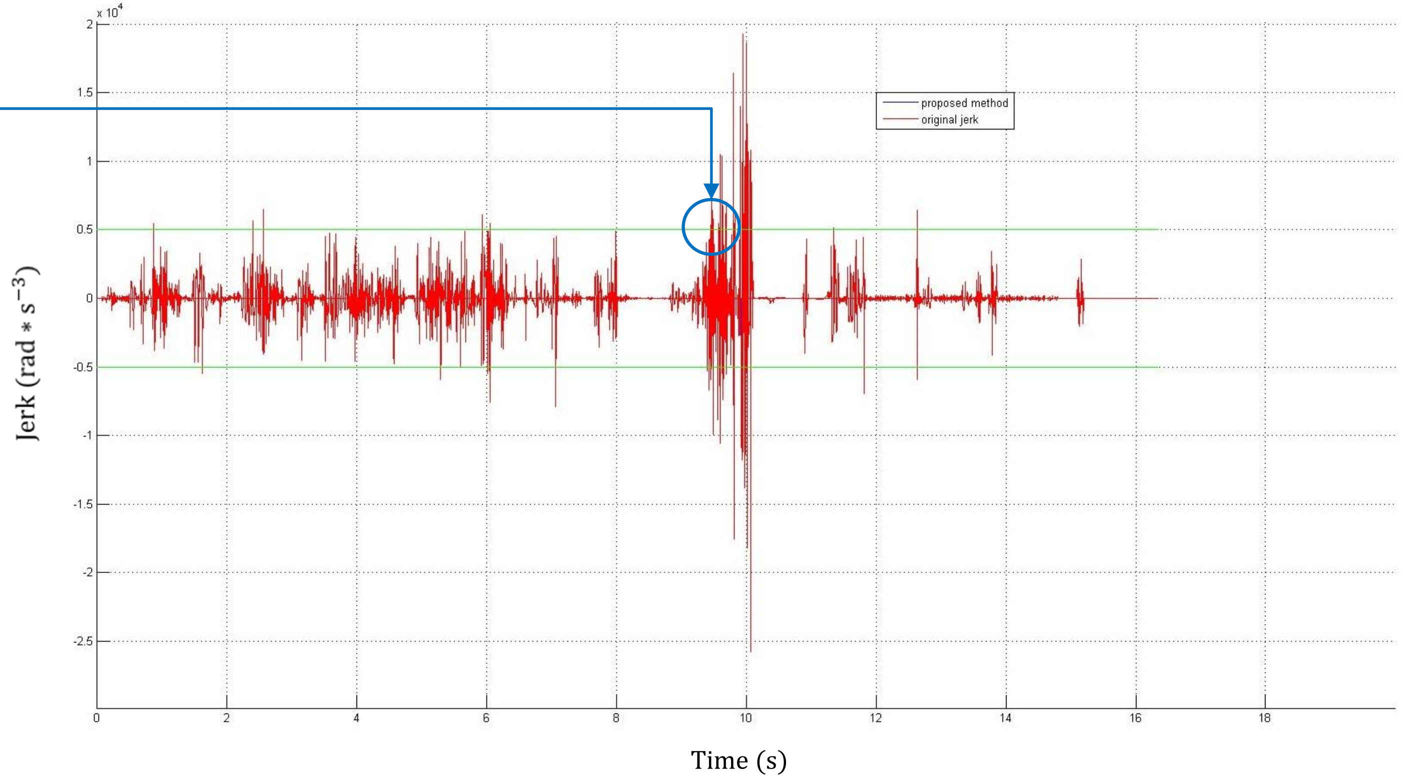


Experimental results



Zoom on jerk limitation effects

Jerk comparison



Conclusion

- Advantage of the method: smooth acceleration trajectories
  - Stability of humanoid robots.
  - Less stress on the joints motors.
- Future work
  - Real-time implementation of the algorithm.
  - Validation on walking patterns while avoiding self-collision.
  - Validation on the real humanoid robot HRP-2