Whole-body dynamic motion planning with centroidal dynamics and full kinematics

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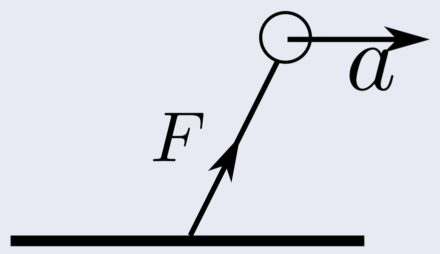
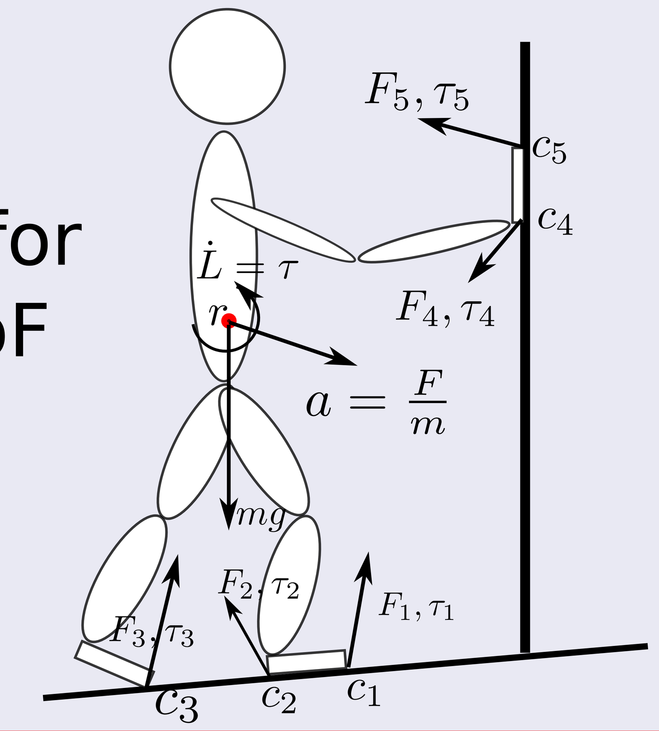
Introduction

Linear Inverted Pendulum
- compute ZMP with point-mass model
- linear system, analytical solution
- co-planar contact
- solve kinematics separately



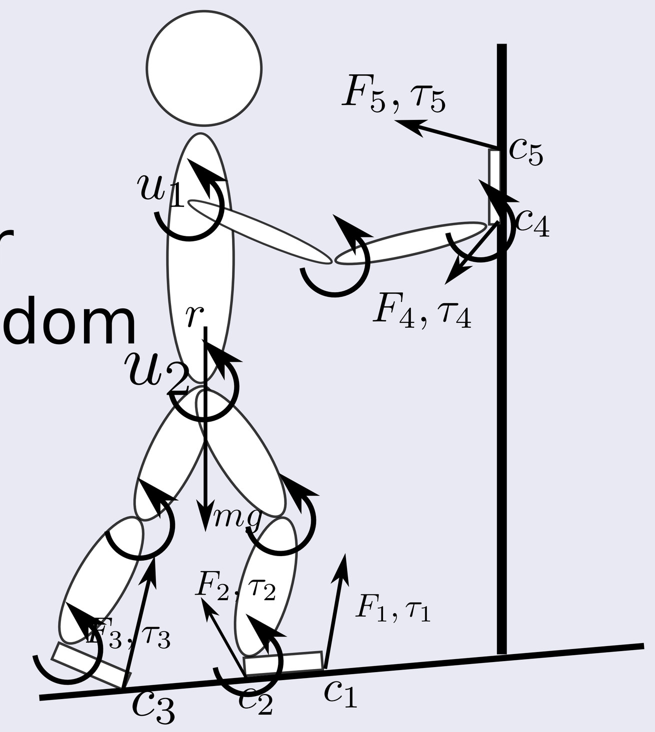
Our approach

dynamic constraint for 6 under-actuated DoF



Full body trajectory optimization

dynamic constraint for EVERY degrees of freedom



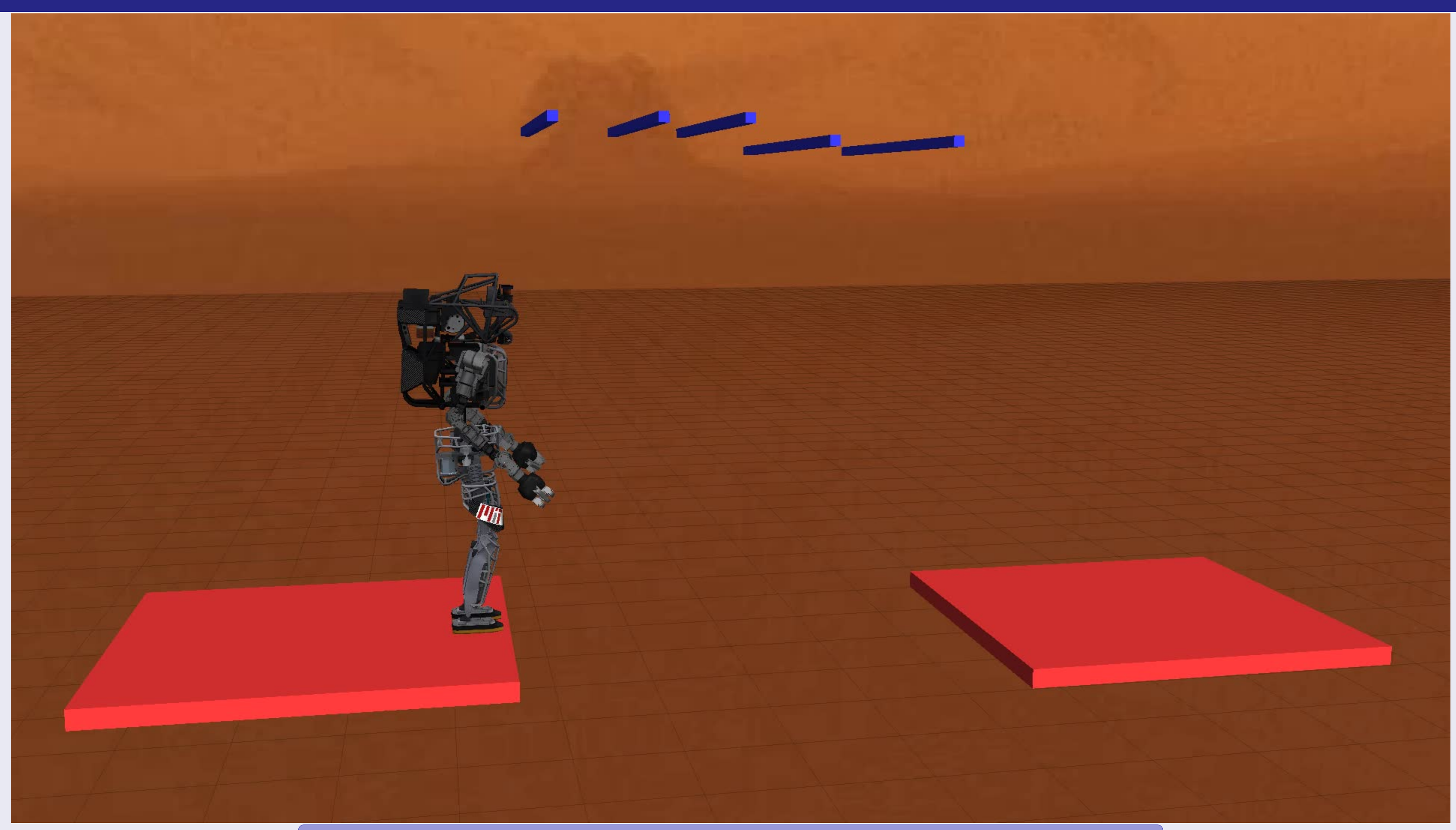
Accuracy → Complexity

Overview

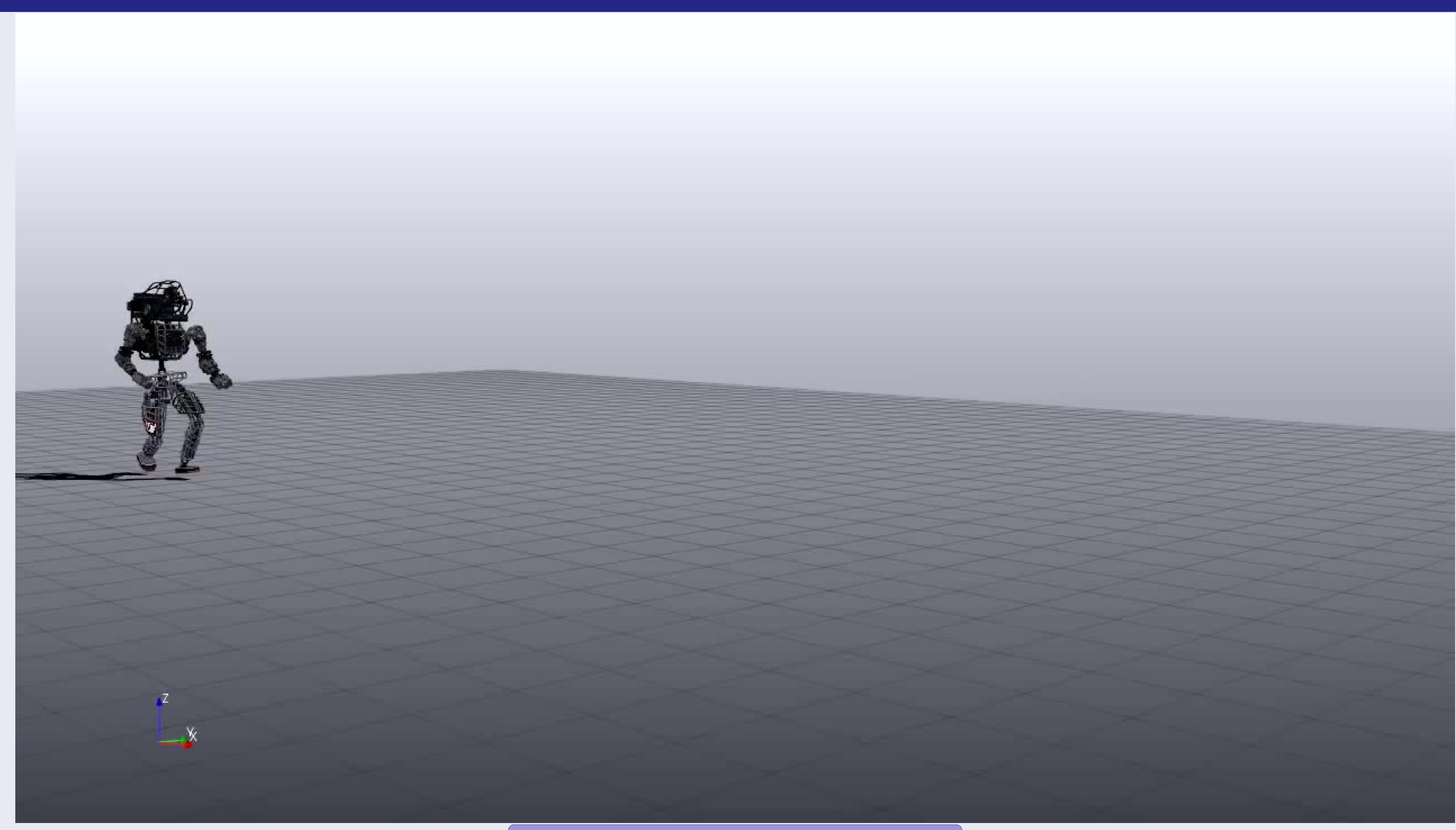
task constraint, whole body robot model, admissible contact regions → nonlinear optimization solver → robot state trajectory, contact wrench profile → inverse dynamics → robot actuator input

Code can be downloaded from Drake `https://drake.mit.edu`

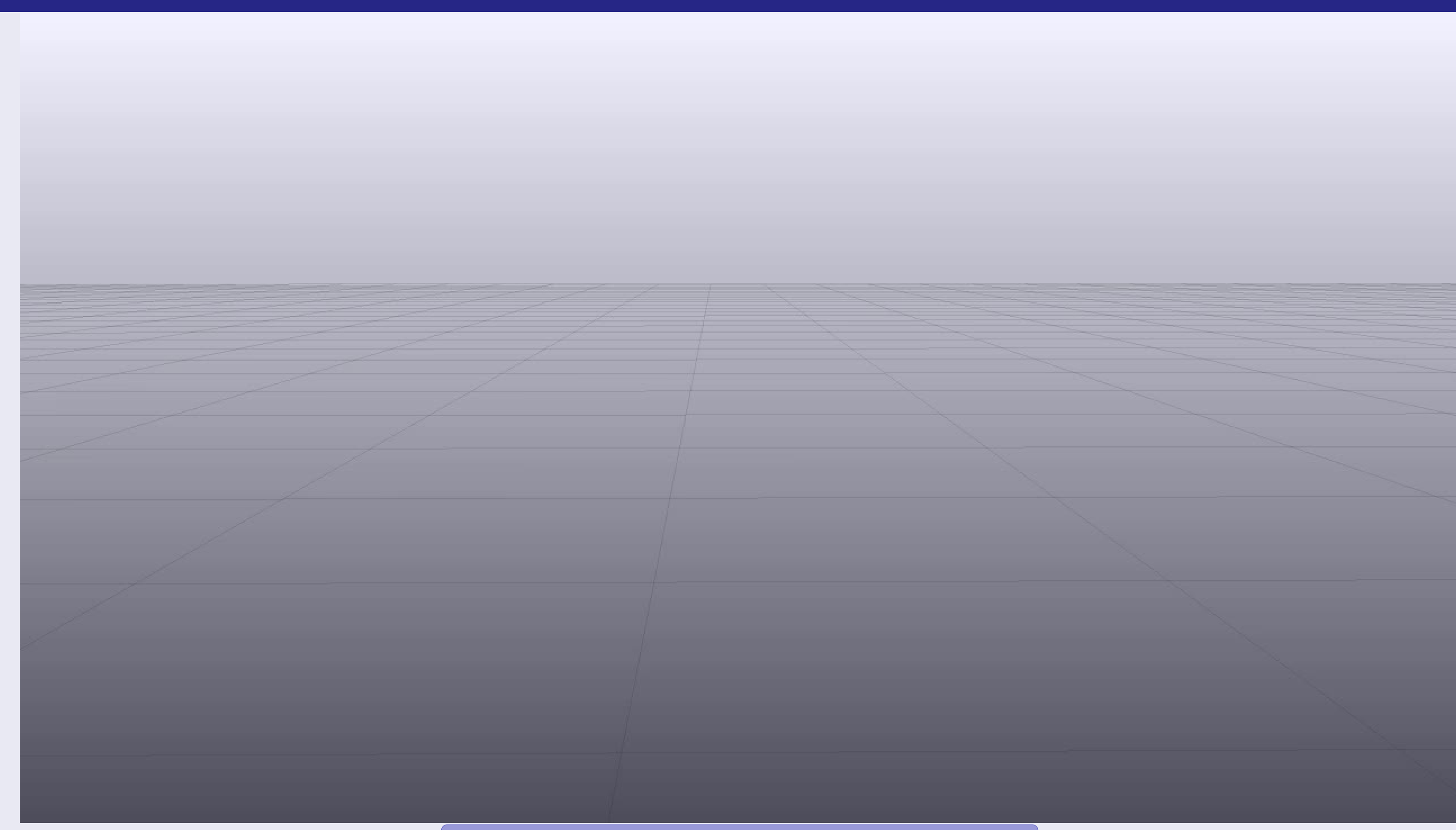
Results



Atlas playing monkey bars

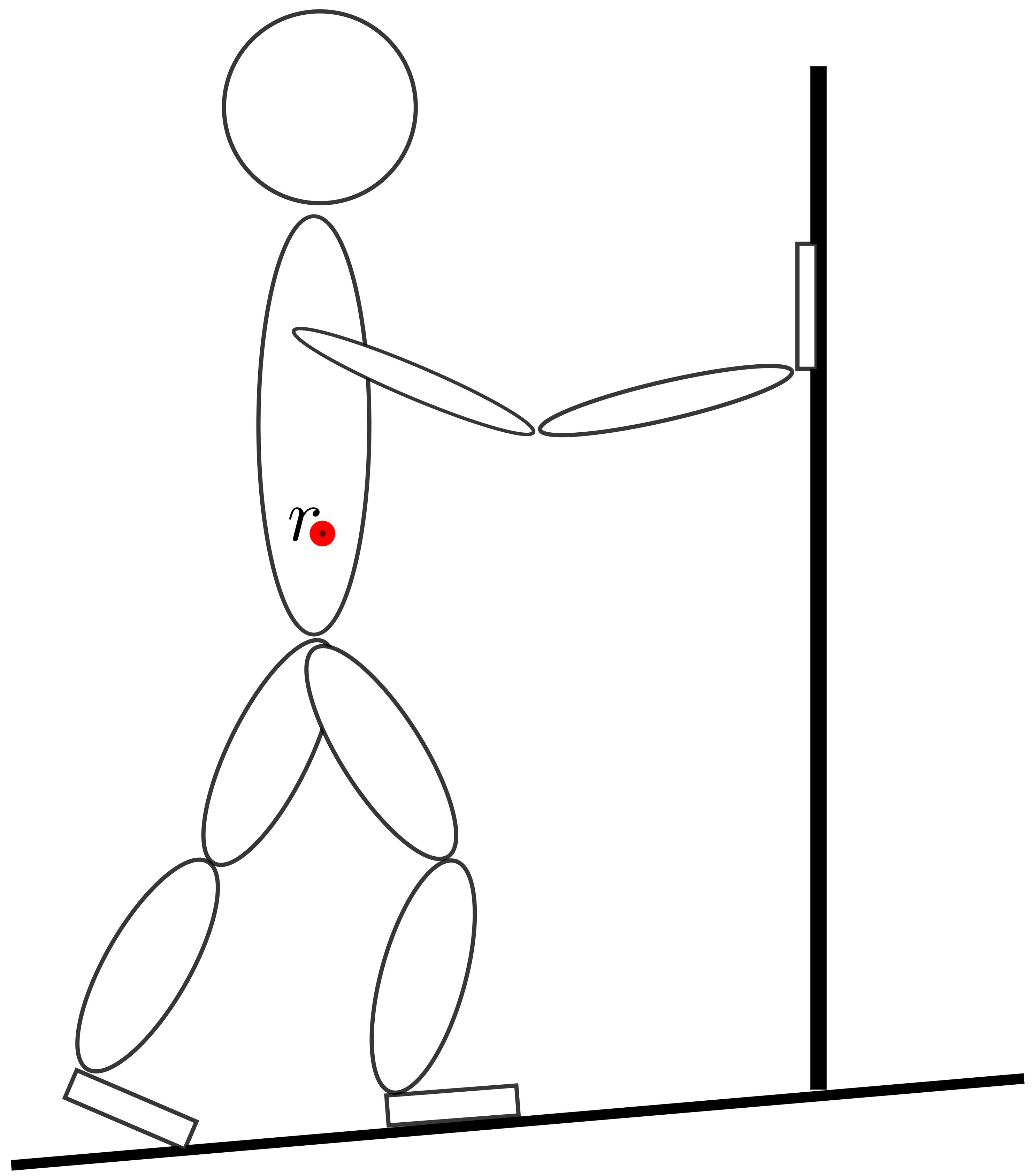


Atlas running



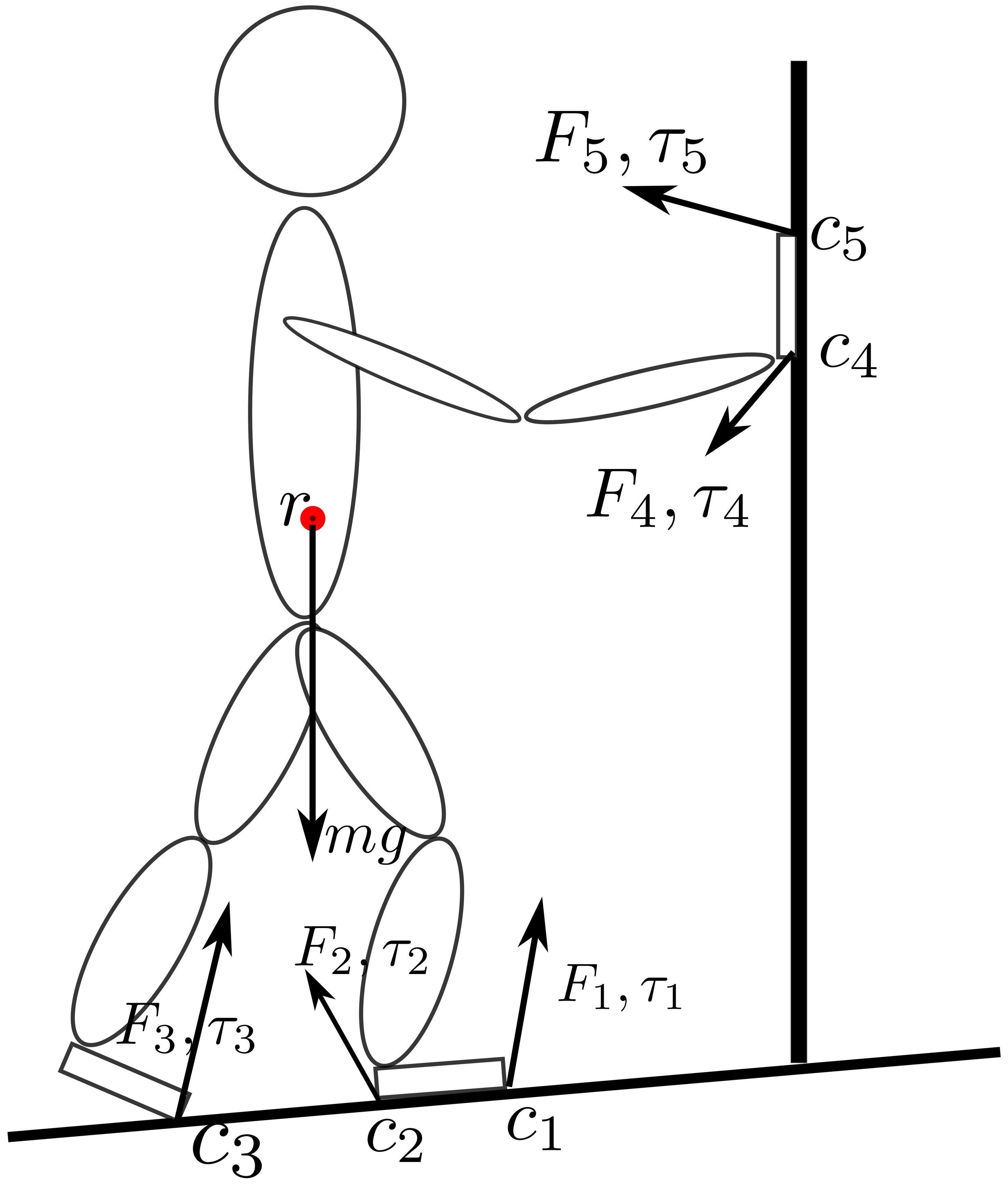
Little Dog running

Consider a robot interacting with the environment, with foot and hand contact. The red dot *r* is the Center of Mass location.



Go back

The robot is in contact with the envinronment at point c_i, subject to contact wrench $[F_i, \tau_i]$ and the gravitational force mg at the CoM.



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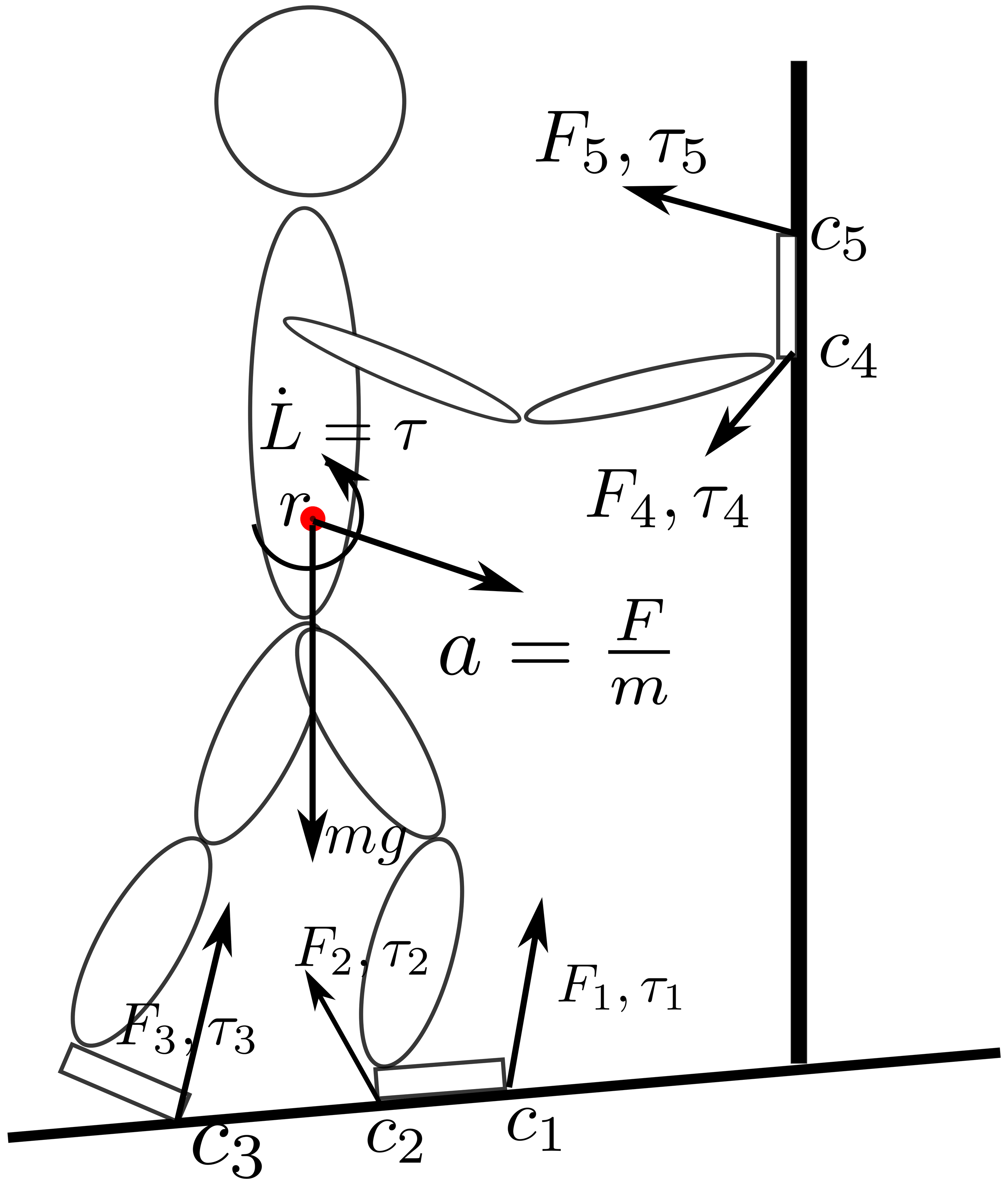
The rate of centroidal linear and angular momentum should equal to the total wrench at the CoM.



The rate of centroidal linear momentum is $m\ddot{r}$. The centroidal angular momentum can be computed from the robot posture and velocity. [D.E.Orin et al]

$$m\ddot{r} = \sum_j F_j + m\mathbf{g} \quad \text{Newton's law on CoM acceleration}$$

$$\dot{L} = \sum_j (c_j - r) \times F_j + \tau_j \quad \text{rate of centroidal angular momentum equals to external torque}$$

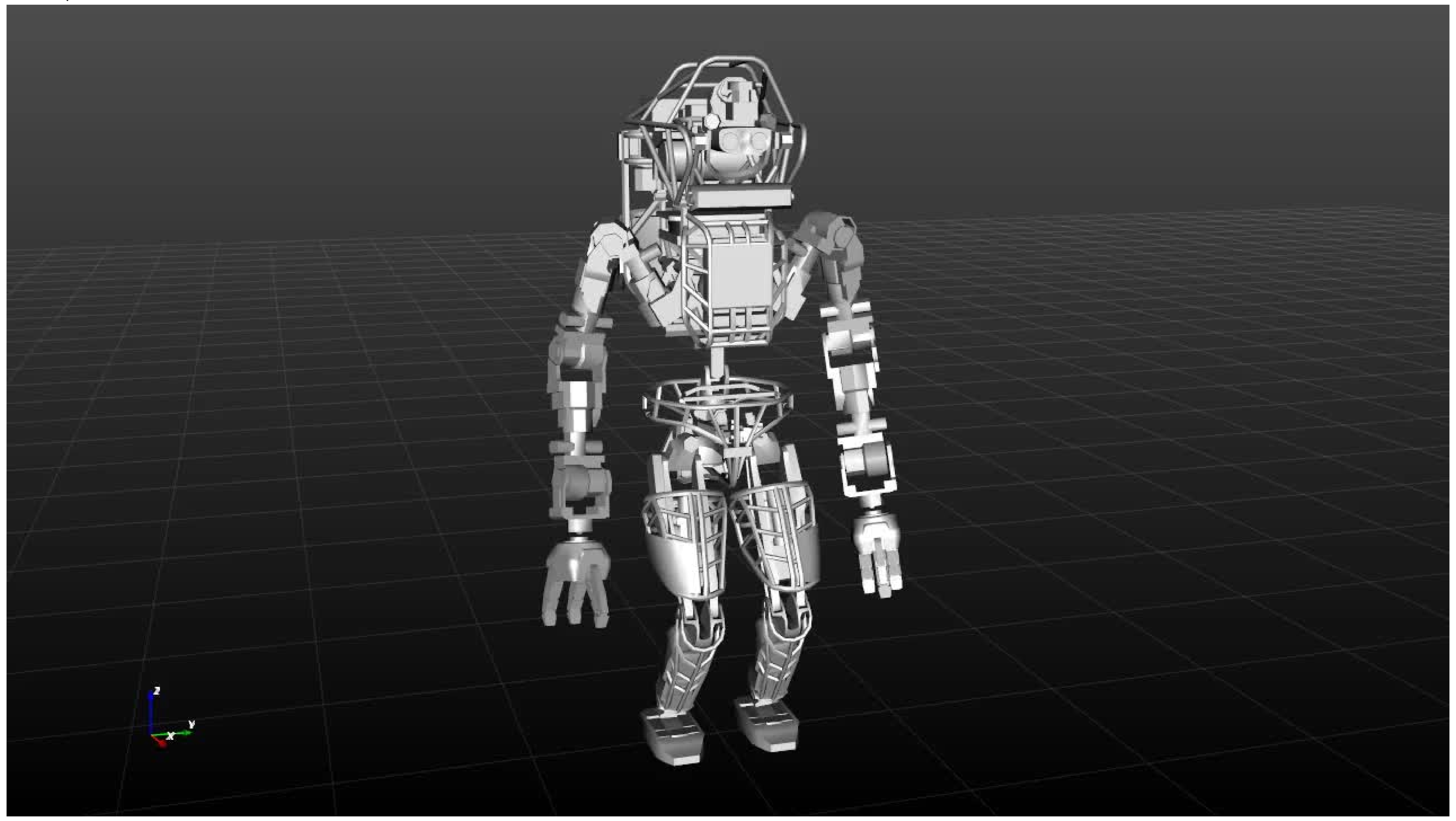
$$r = com(q) \quad \text{compute CoM from posture}$$

$$L = A(q)v \quad \text{compute angular momentum from robot state}$$

Go back

Accommodate a variety of kinematic constraints

- Position of an end-effector
- Orientation of an end-effector
- Gaze at a point
- Collision avoidance
- Quasi-static



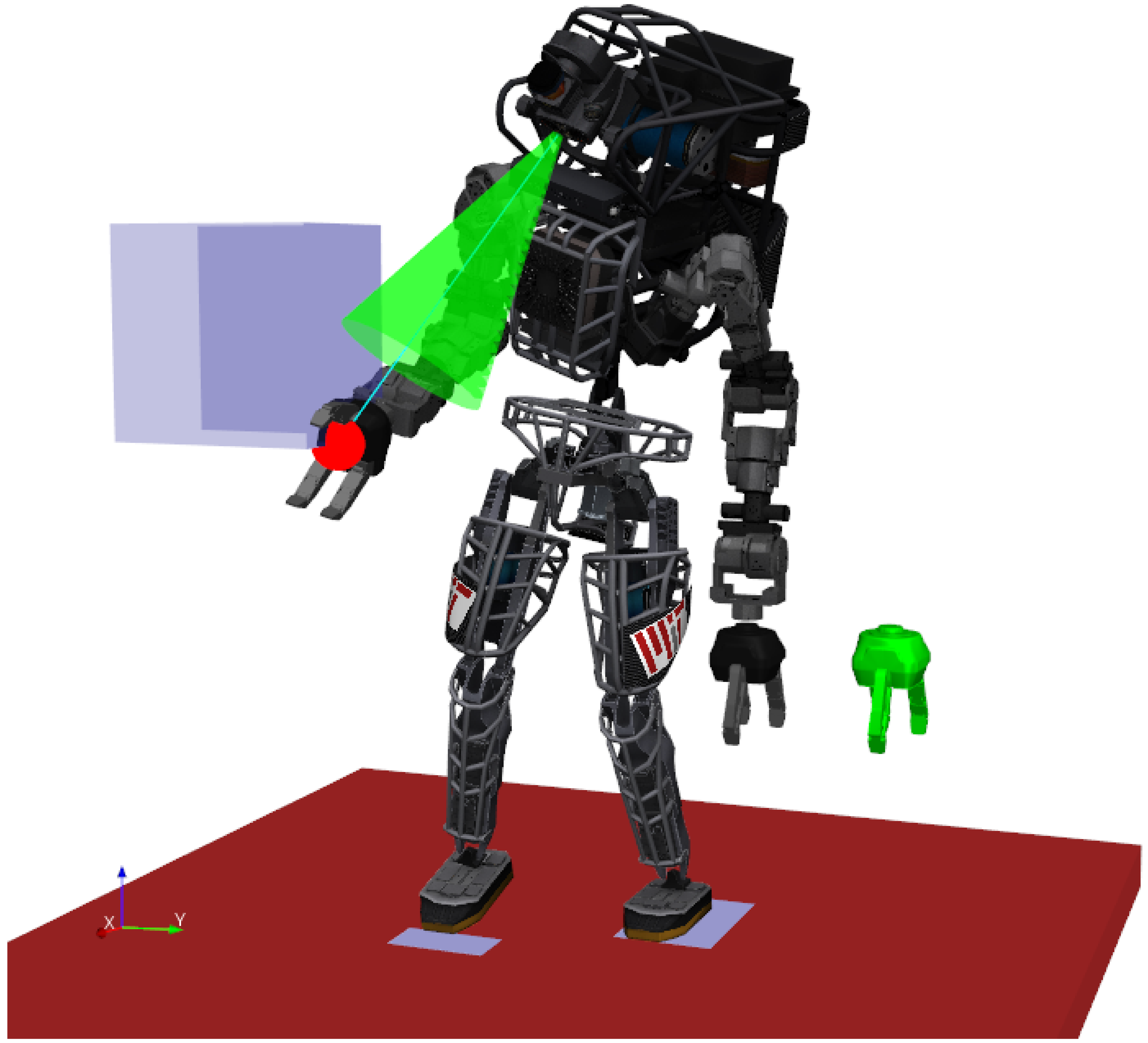


Figure: Solving inverse kinematics problem with different types of kinematic constraints.

Go back

Hard to specify the contact sequence when mulitple contact points can be active. Exploit the complementarity constraint on normal contact force F^n and distance to contact $\phi(c)$. [M.Posa]

$$
\begin{aligned}
< \phi_j(\mathbf{c}_j), F_j^n >= 0 \\
\phi_j(\mathbf{c}_j) \geq 0 \\
F_j^n \geq 0
\end{aligned}
$$

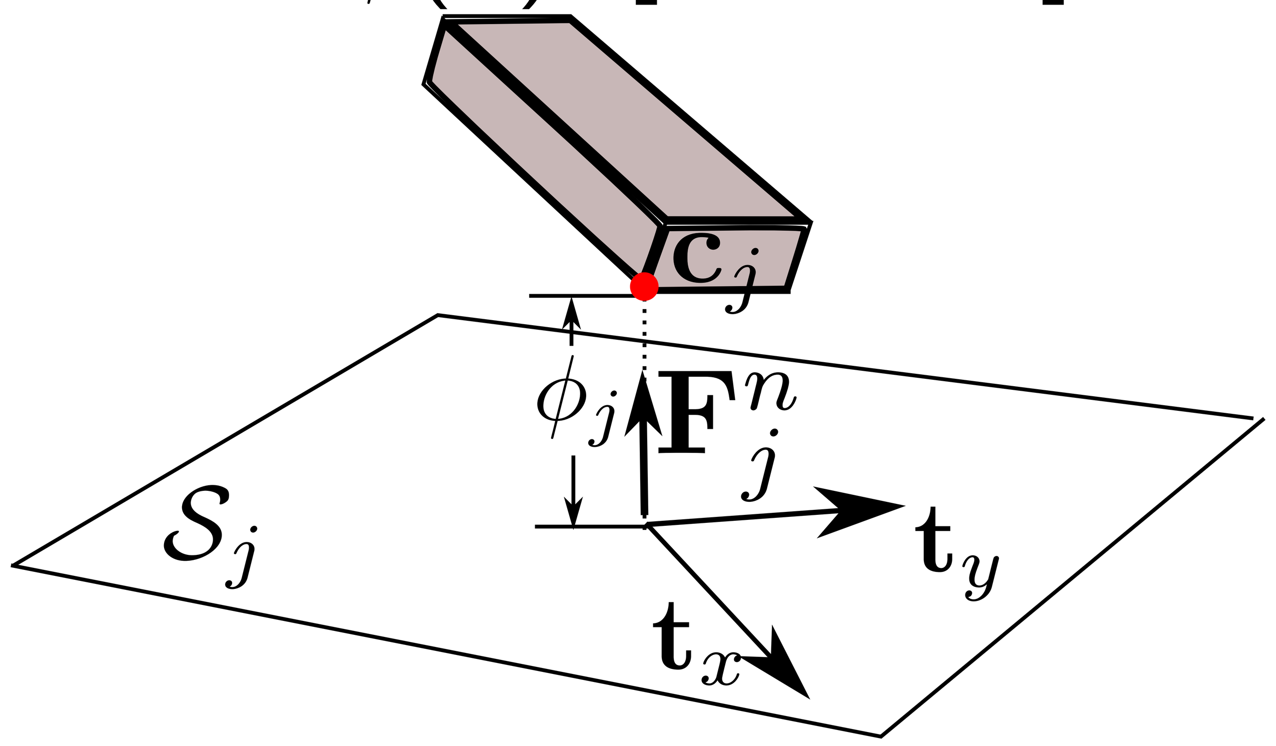
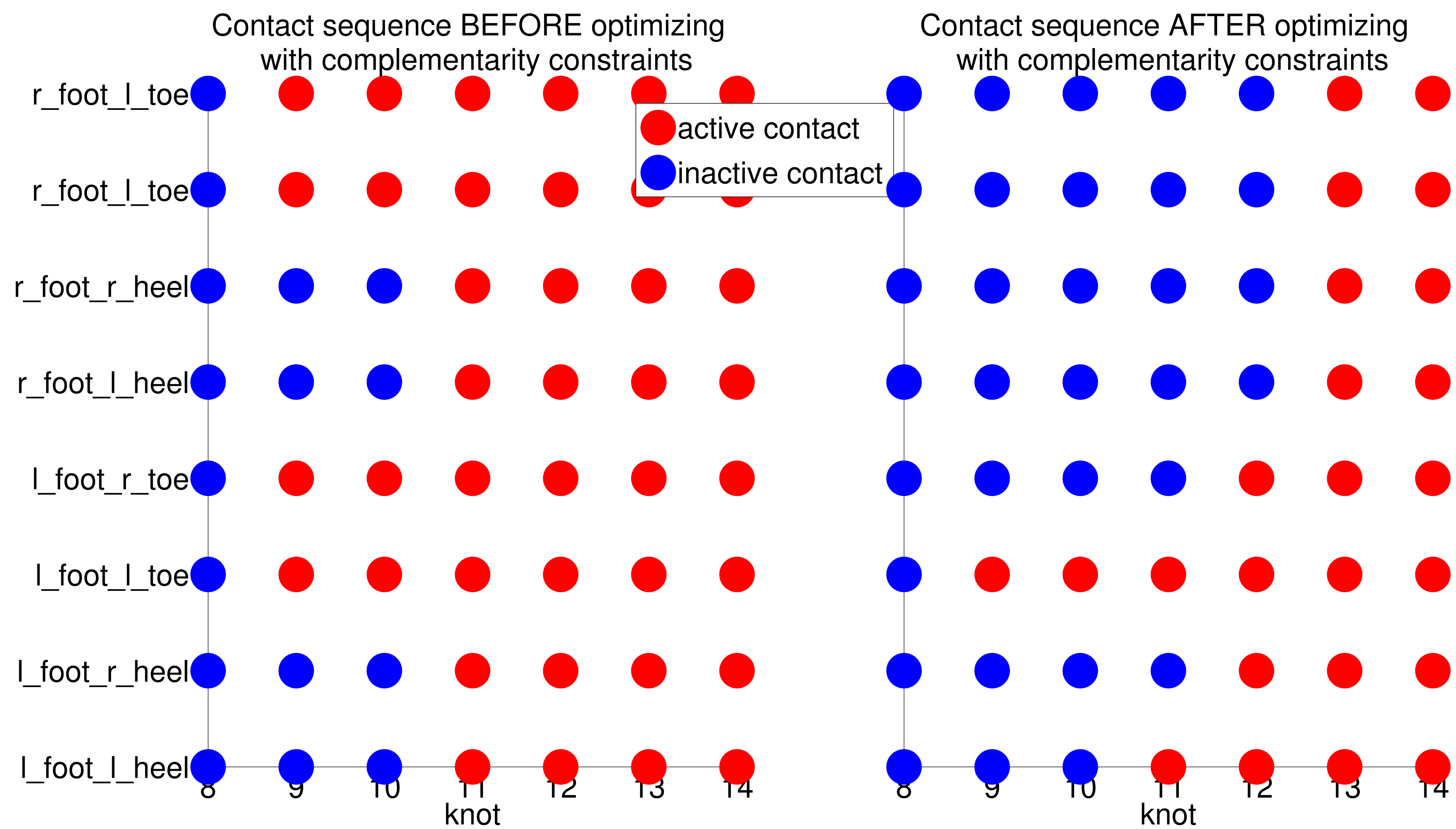


Figure: Illustration of the contact point $\mathbf{c}_j$, its distance ϕ_j to the contact surface $\mathcal{S}_j$, and the local coordinate frame on the tangential surface, with unit vector $\mathbf{t}_x, \mathbf{t}_y$. The complementarity condition holds between contact distance ϕ_j and the normal contact force $\mathbf{F}_j^n$.



Go back

- Sample the whole trajectory into N knot points.

Go back

1. Sample the whole trajectory into N knot points.
2. In k^{th} point, assign the posture $\mathbf{q}[k]$, velocity $\mathbf{v}[k]$, contact wrench $\mathbf{F}[k], \tau[k]$ and time duration $h[k]$ as decision variables.

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$$\min_{\substack{\mathbf{q}[k],\mathbf{v}[k],h[k]\\ \mathbf{r}[k],\dot{\mathbf{r}}[k],\ddot{\mathbf{r}}[k]\\ \mathbf{c}_j[k],\mathbf{F}_j[k],\tau_j[k]\\ \mathbf{L}[k],\dot{\mathbf{L}}[k]}} \sum_{k=1}^{N} \left(|\mathbf{q}[k]-\mathbf{q}_{\mathbf{nom}}[k]|_{Q_q}^2 + |\mathbf{v}[k]|_{Q_v}^2 + |\ddot{\mathbf{r}}[k]|^2 + \sum_j (c_1 \left|\mathbf{F}_j[k]\right|^2 + c_2|\tau_j[k]|^2) \right) h[k]$$

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$$\text{s.t Dynamic constraint}\begin{cases} m\ddot{\mathbf{r}}[k]=\sum_j \mathbf{F_j}[k]+m\mathbf{g}\\ \dot{\mathbf{L}}[k]=\sum_j(\mathbf{c_j}[k]-\mathbf{r}[k])\times\mathbf{F_j}[k]+\tau_\mathbf{j}[k]\\ \mathbf{L}[k]=\mathbf{A}(\mathbf{q}[k])\mathbf{v}[k]\\ \mathbf{r}[k]=com(\mathbf{q}[k])\end{cases}$$

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$$\text{Backward-Euler integration} \begin{cases} \mathbf{q}[k] - \mathbf{q}[k-1] = \mathbf{v}[k]h[k] \\ \mathbf{L}[k] - \mathbf{L}[k-1] = \dot{\mathbf{L}}[k]h[k] \end{cases}$$

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$$\text{Quadratic interpolation on CoM} \begin{cases} \mathbf{r}[k] - \mathbf{r}[k-1] = \frac{\dot{\mathbf{r}}[k]+\dot{\mathbf{r}}[k-1]}{2}h[k] \\ \dot{\mathbf{r}}[k] - \dot{\mathbf{r}}[k-1] = \ddot{\mathbf{r}}[k]h[k] \end{cases}$$

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s.t Dynamic constraint
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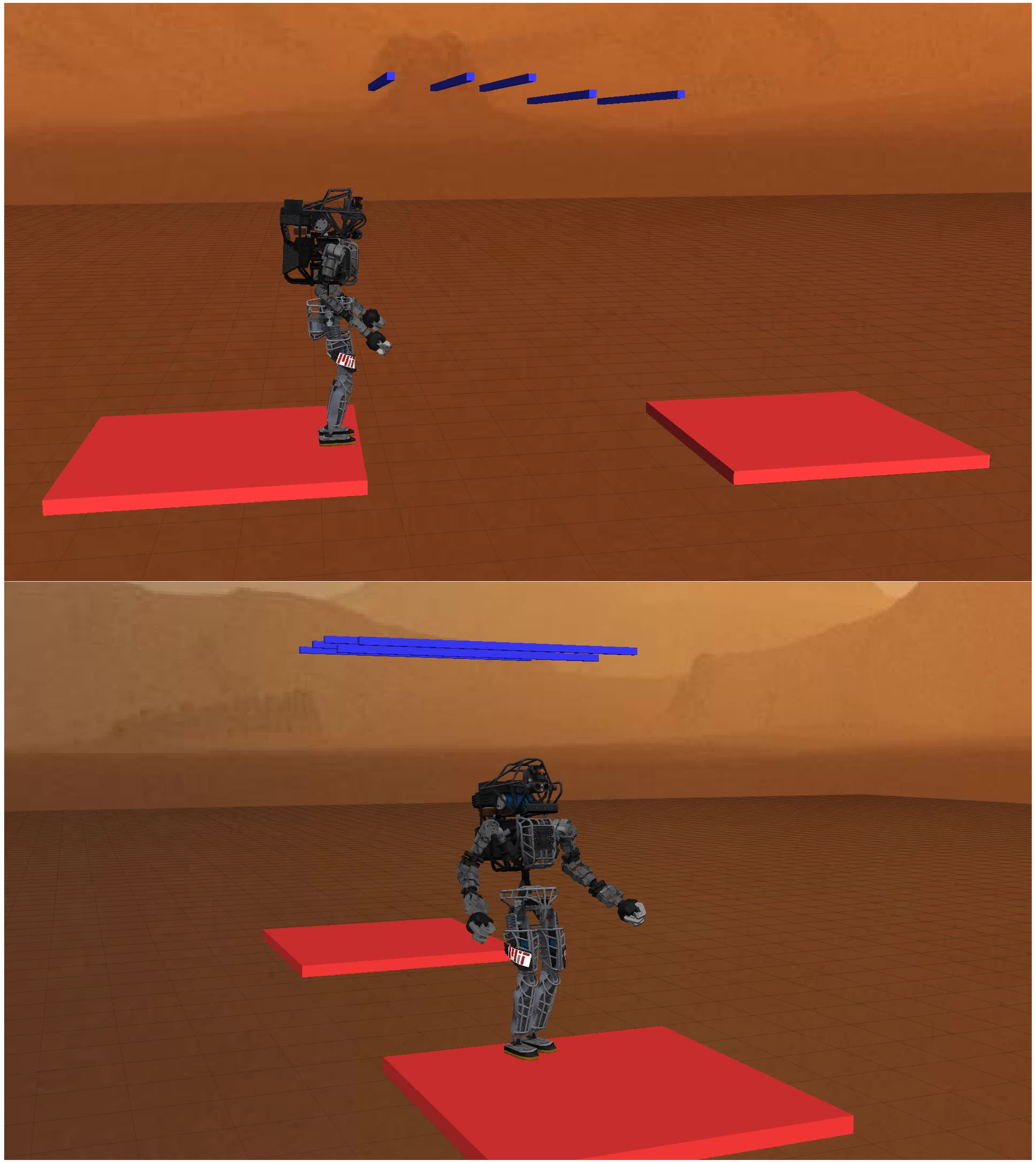
Kinematic constraint for contact
$$\begin{cases} \mathbf{c}_j[k] = p_j(\mathbf{q}[k]) \\ \mathbf{c}_j[k] \in \mathcal{S}_j[k] \end{cases}$$

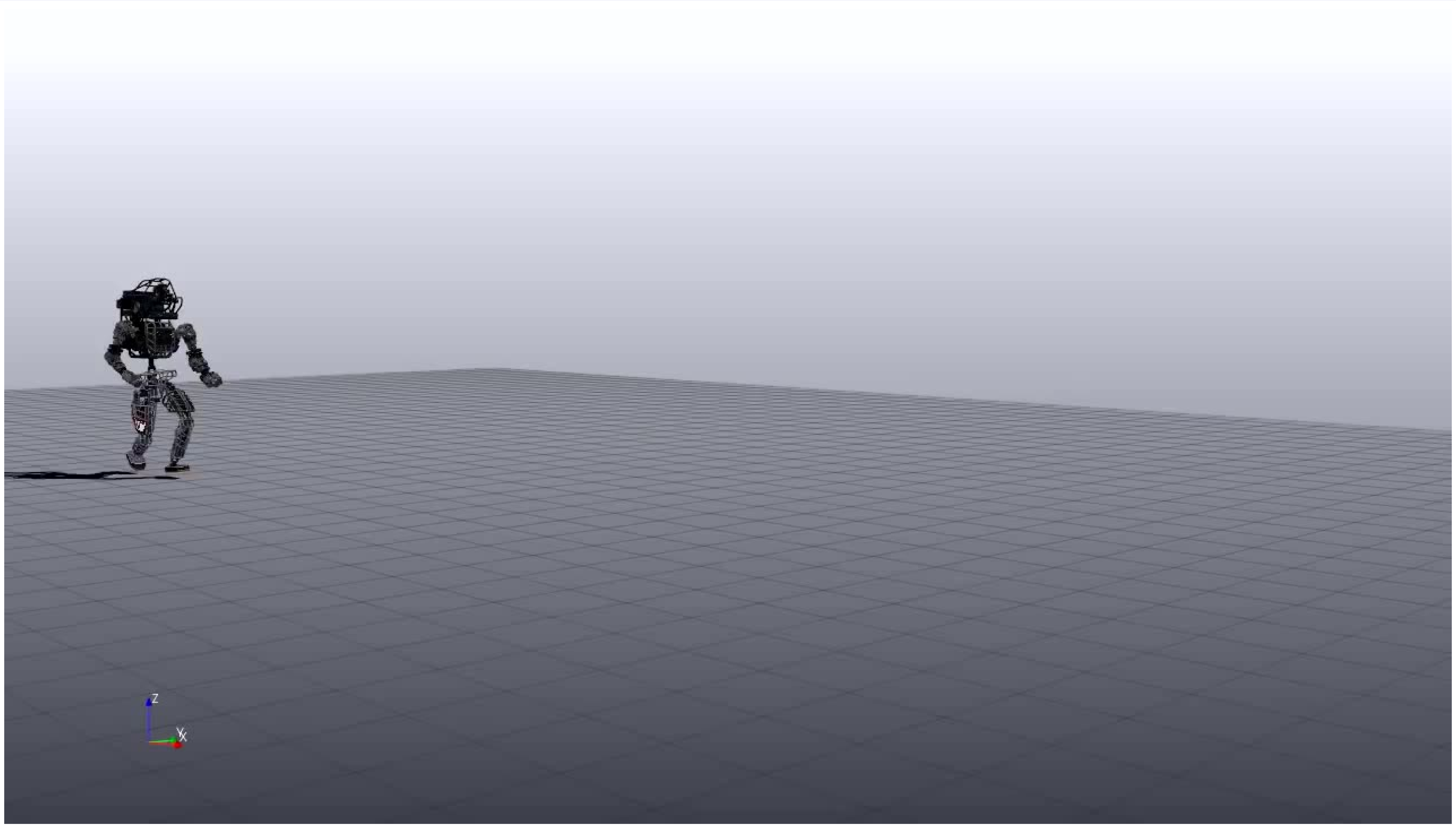
Go back

Monkey bars



Go back

Atlas running



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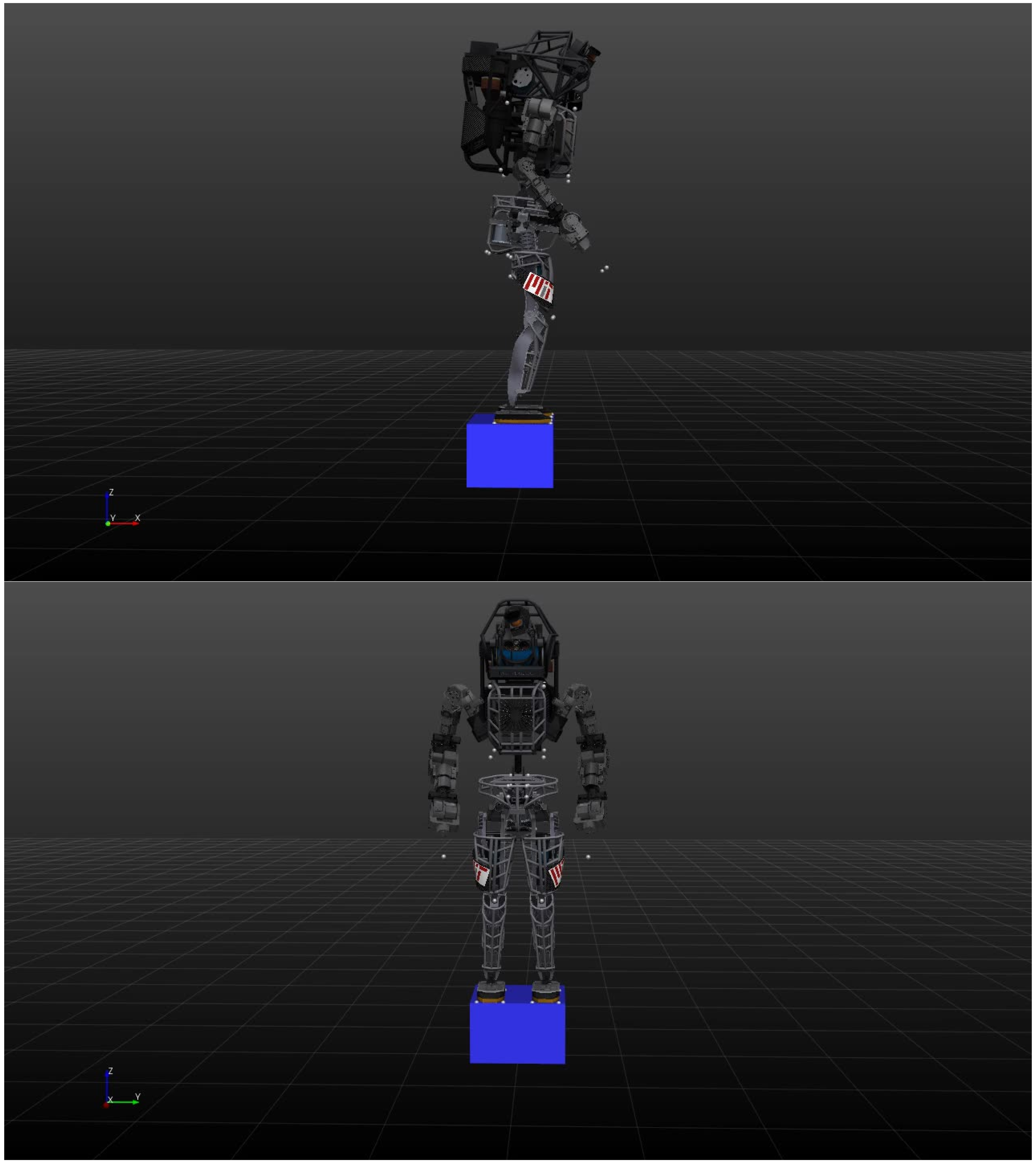
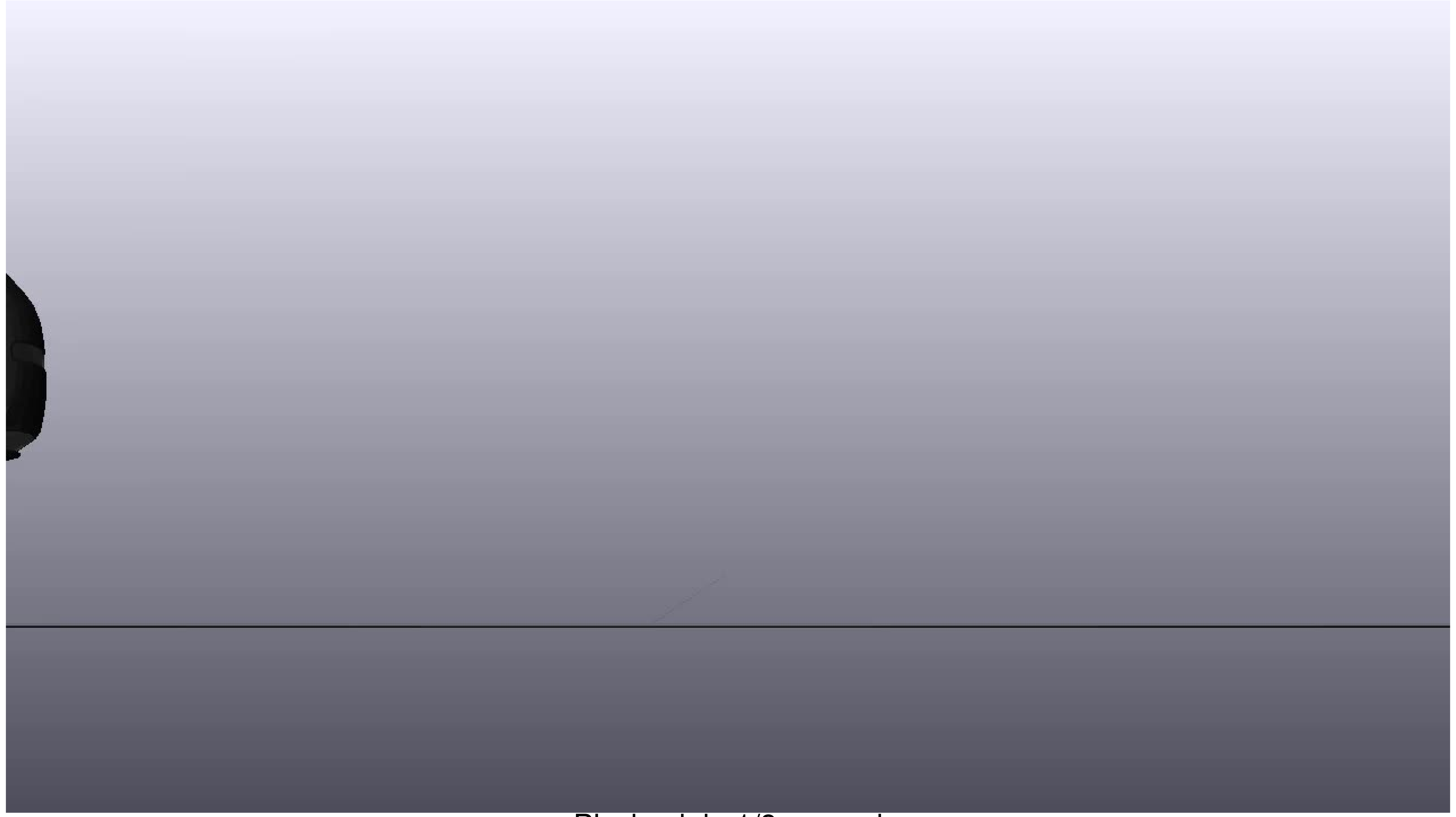
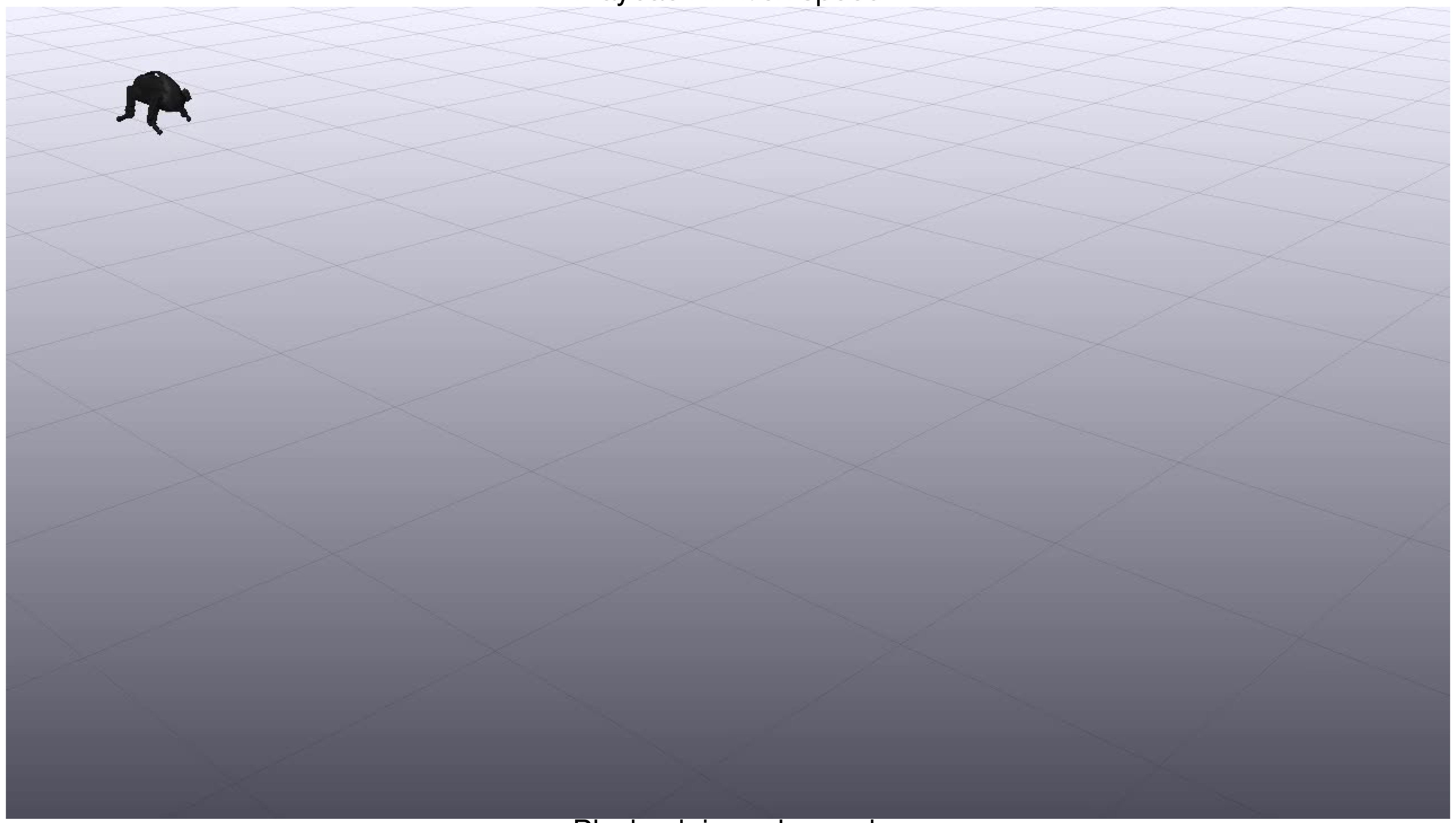


Go back

Little dog running

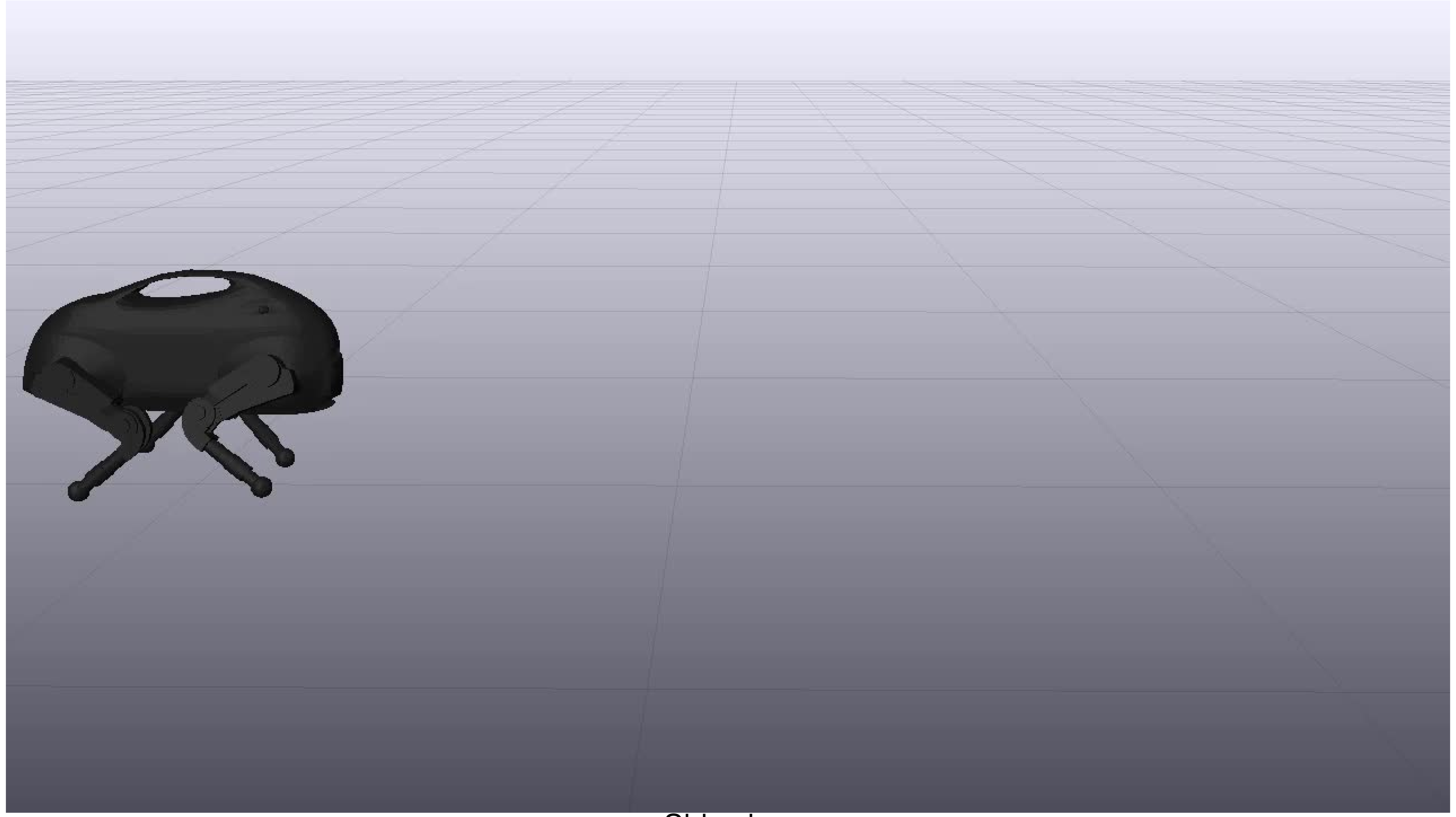


Playback in 1/8x speed

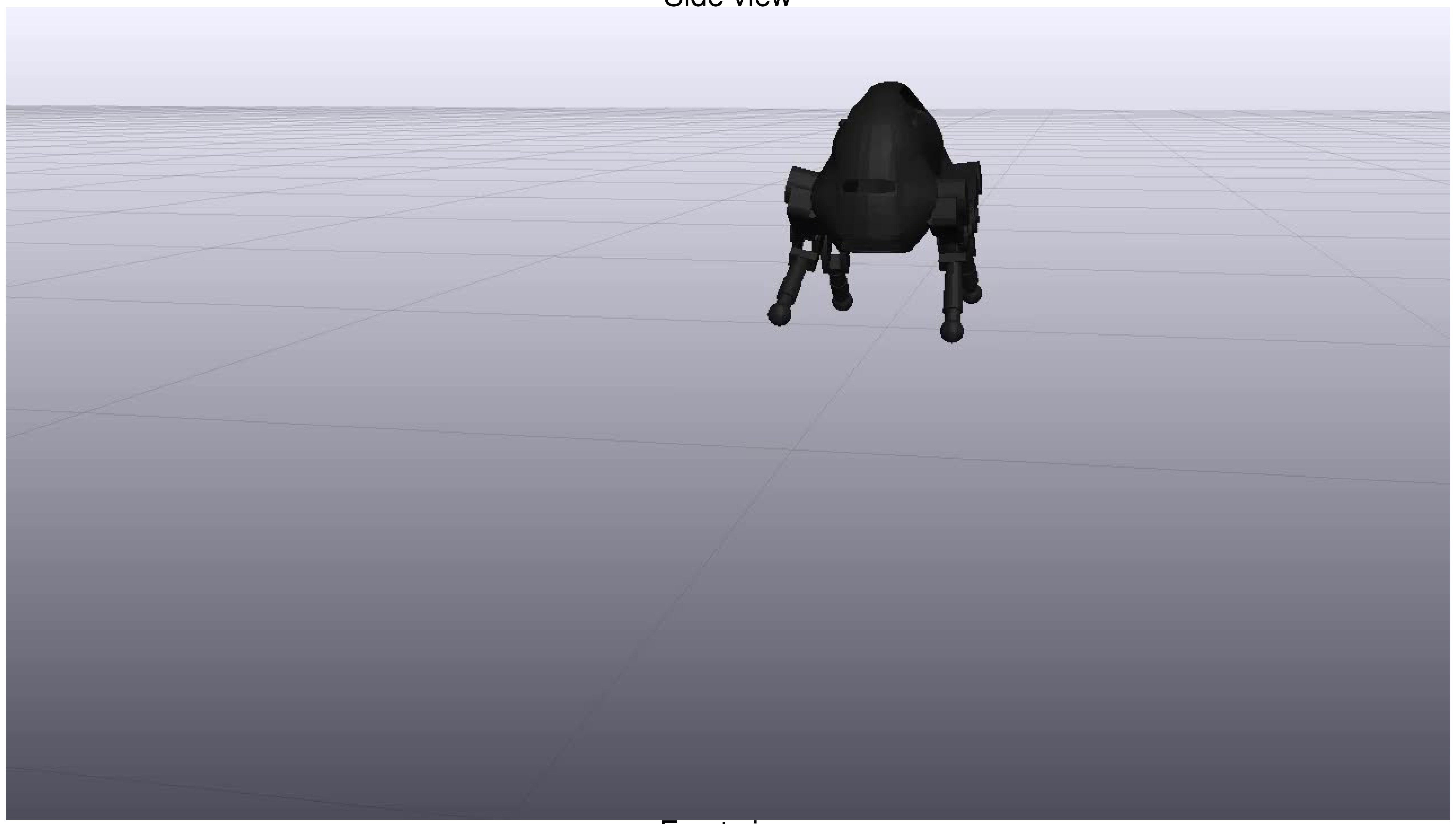


Playback in real speed.

Go back

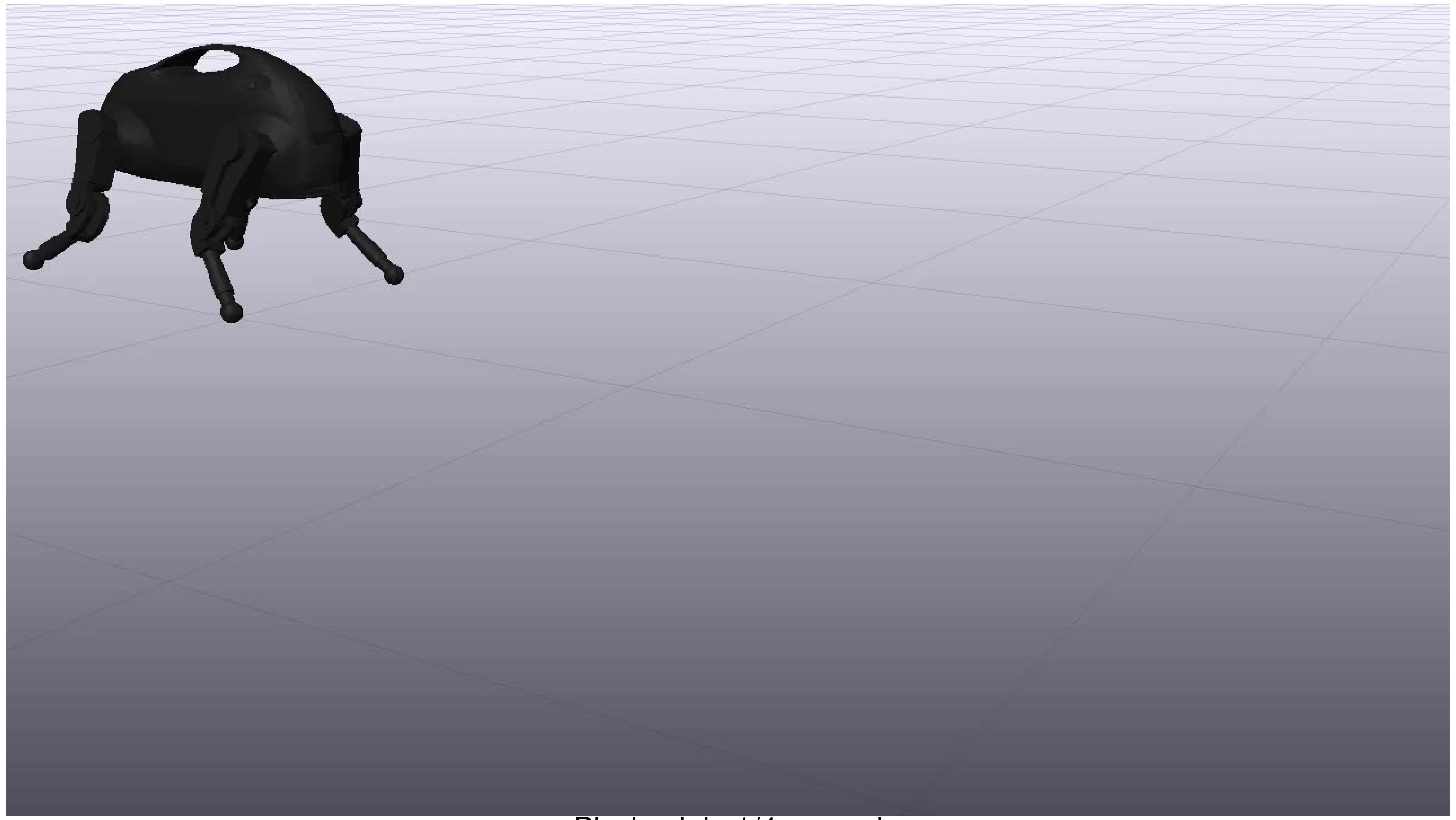


Side view

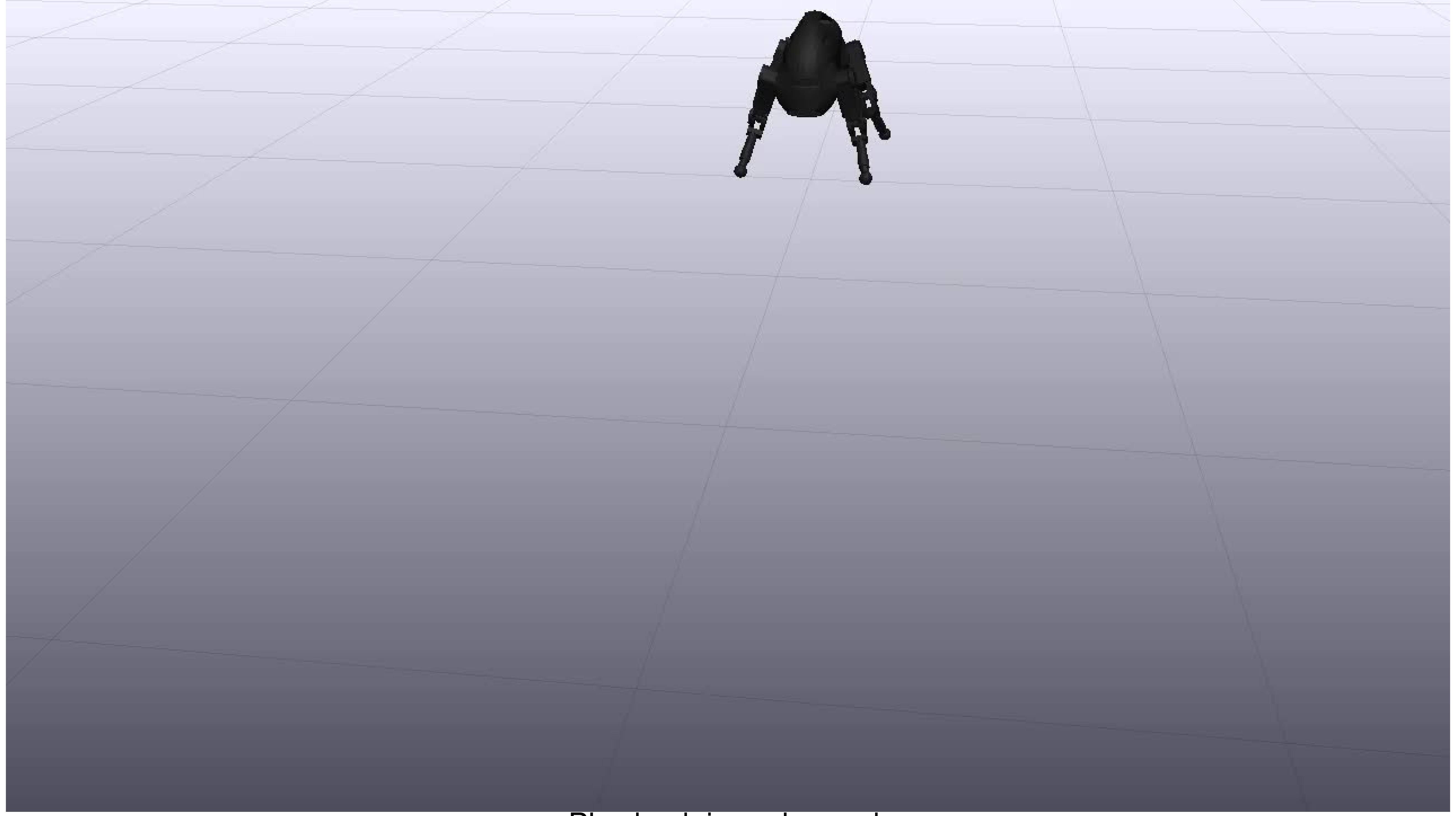


Front view

Go back

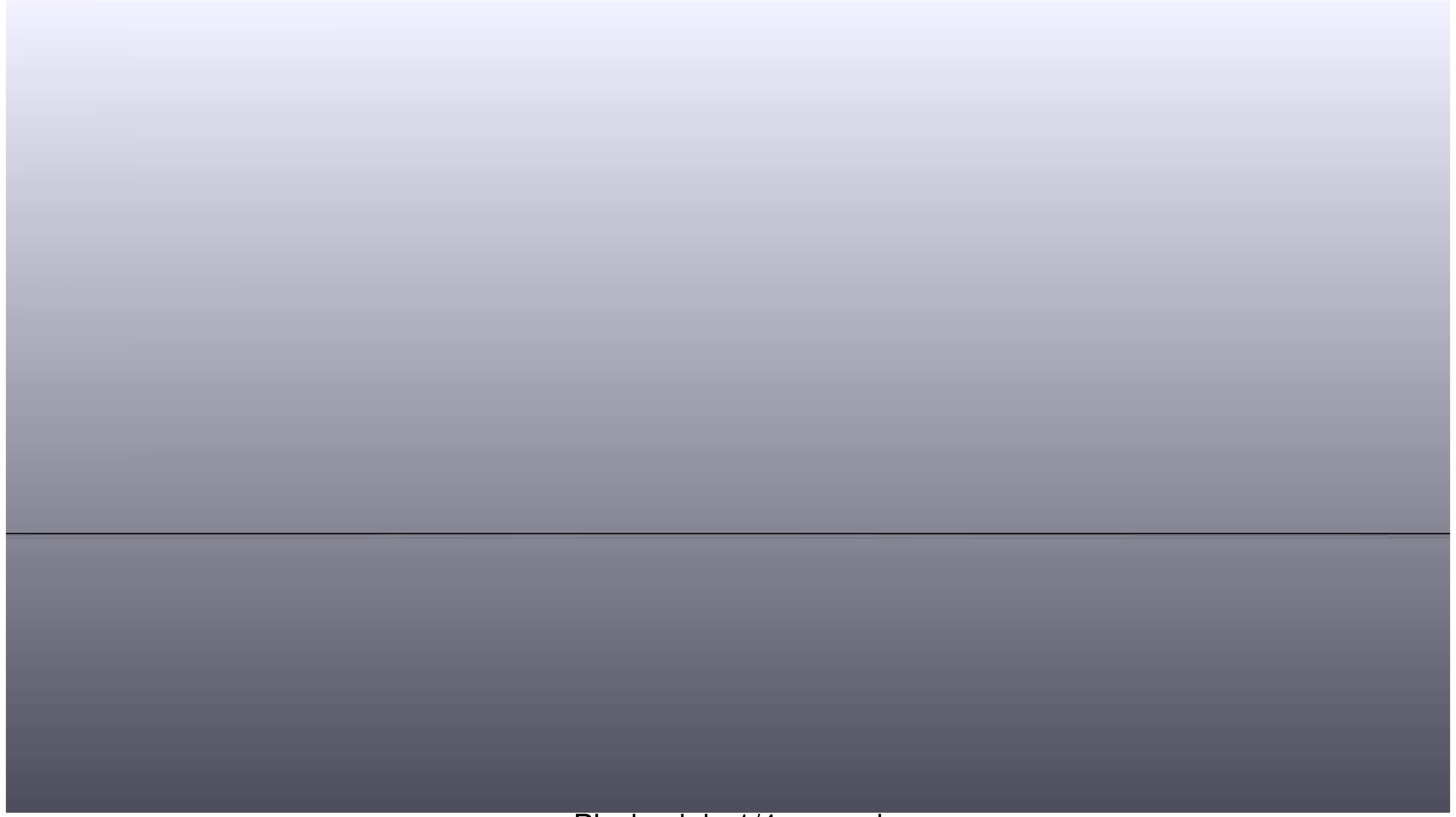


Playback in 1/4x speed

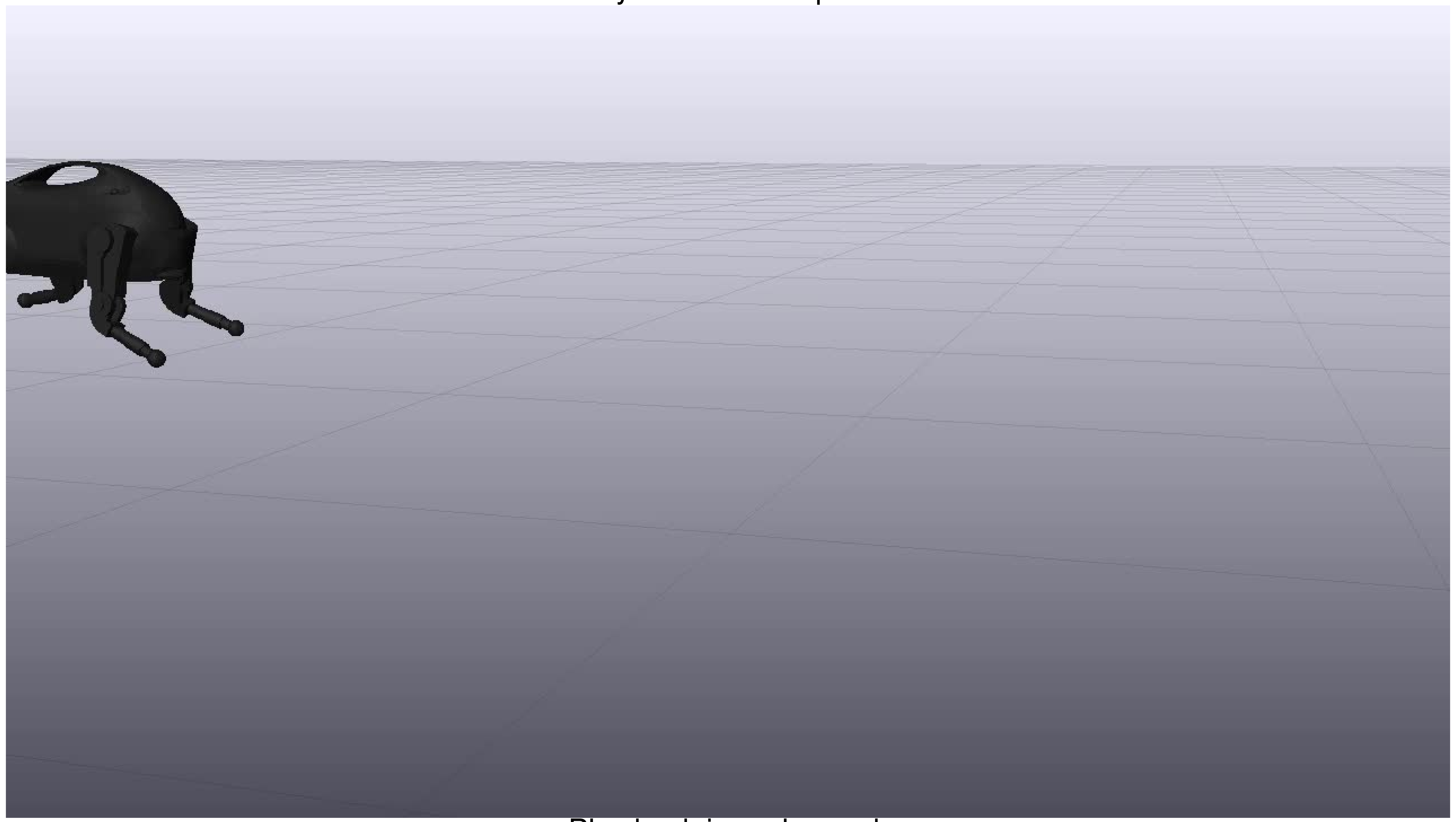


Play back in real speed.

Go back



Playback in 1/4x speed



Play back in real speed.

Go back