Inverse Kinematics: New Method for Minimum Jerk Trajectory Generation

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***Abstract*— The problem of inverse kinematics is revisited in the present paper. The paper is focusing on the problem of solving the inverse kinematics problem while minimizing the jerk of the joints trajectories. Even-though the conventional inverse kinematics algorithms have been proven to be efficient in many applications, no contraintes on the accelerations or the jerk of the obtained trajectories can be guaranteed. The proposed algorithm yields smooth velocity and acceleration trajectories, which are mostly important in the case of humanoid robots as the balance is directly related to the joint accelerations. The algorithm is using the joints jerk as the control parameter instead of the classical use of the joints velocity, as a result constraints of the jerk function can be easily incorporated.**

To validate the proposed approach, a simulation scenario of a boxing motion with the humanoid robot HRP-2 has been conducted. The simulation results have revealed the effectiveness of the proposed method to solve the inverse kinematics problem while considering the joint jerks efficiently.

I. Inverse Kinematics Problem: Revisited

The inverse kinematics problem is one of the most studied problems in robotics. Many efficient and robustness algorithms have been proposed for solving the inverse kinematics problem [1], [2].

The inverse kinematics problem was originally formulated as follows:

$$\begin{aligned} &\min_{\dot{q}} \dot{q_t}^T Q \dot{q_t} \\ &\text{subject to} \\ &J\dot{q_t} = \dot{r_t} \end{aligned} \tag{1}$$

where Q is a diagonal and semi-positive matrix, J is the Jacobian matrix, $\dot{q}$ is the joint velocity and $\dot{r_t}$ is the velocity of the end-effector.

The optimization problem Eq. (1) can be efficiently solved using the pseudo-inverse technique [1], [2]. However, in order to consider the velocity and joints limits as well as avoiding obstacles, an inequality constraints should be added. The general inverse kinematics problem is therefore can be formulated as fellows:

$$\begin{aligned} &\min_{\dot{q}} \dot{q_t}^T Q \dot{q_t} \\ &\text{subject to} \\ &J\dot{q_t} = \dot{r_t} \\ &A\dot{q_t} \leq b \end{aligned} \tag{2}$$

To solve the optimization problem Eq. (2), a Quadratic Programming (QP) solver can be efficiently used.

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II. Motivation

Many modern humanoid robots are using the principle of Zero Moment Point (ZMP) [3] to generate stable motion and walking patterns. A trajectory is dynamically stable if the ZMP is always inside of the polygon of support.

Let the ZMP on the horizontal ground be given by the following vector

$$\boldsymbol{p} = \begin{bmatrix} p_x & p_y \end{bmatrix}^T \tag{3}$$

To compute $\boldsymbol{p}$, one can use the following formula

$$\boldsymbol{p} = \boldsymbol{N} \ \frac{\boldsymbol{n} \times \boldsymbol{\tau}^o}{(\boldsymbol{f}^o|\boldsymbol{n})} \tag{4}$$

where the operator $\times$ and $(.|.)$ refer to the cross and scalar products respectively, and

- $\boldsymbol{N}$ is a constant matrix

$$\boldsymbol{N} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{5}$$

.

- the vector $\boldsymbol{n}$ is the normal vector on the horizontal ground $\left(\boldsymbol{n} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T\right)$.
- The vector $\boldsymbol{f}^o$ is the result of the gravity and inertia forces

$$\boldsymbol{f}^o = M\boldsymbol{g} - \sum_{i=0}^{n} m_i \ \ddot{\boldsymbol{X}}^{c_i} \tag{6}$$

  where $\boldsymbol{g}$ denotes the acceleration of the gravity ($\boldsymbol{g} = -g\boldsymbol{n}$), and M is the total mass of the humanoid robot. The quantities m_i, $\ddot{\boldsymbol{X}}^{c_i}$ are the mass of the i^{th} link and the acceleration of its center of mass c_i respectively. Note that m_0, $\ddot{\boldsymbol{X}}^{c_0}$ are, respectively, the mass and the acceleration of the free-flyer joint (pelvis joint) of the humanoid robot.
- $\boldsymbol{\tau}^o$ denotes the moment of the force $\boldsymbol{f}^o$ about the origin of the fixed world frame. The expression of $\boldsymbol{\tau}^o$ is the following

$$\boldsymbol{\tau}^o = \sum_{i=0}^{n} \left(m_i \ \boldsymbol{X}^{c_i} \times \left(\boldsymbol{g} - \ddot{\boldsymbol{X}}^{c_i} \right) - \dot{\boldsymbol{\mathcal{L}}}^{c_i} \right) \tag{7}$$

  where $\boldsymbol{\mathcal{L}}^{c_i}$ is the angular momentum at the point c_i

$$\dot{\boldsymbol{\mathcal{L}}}^{c_i} = \boldsymbol{R}^i \left(\boldsymbol{I}_{c_i} \dot{\boldsymbol{\omega}}^i - \left(\boldsymbol{I}_{c_i} \boldsymbol{\omega}^i \right) \times \boldsymbol{\omega}^i \right) \tag{8}$$

  $\boldsymbol{R}^i$ and $\boldsymbol{I}_{c_i}$ are the rotation matrix associated to the i^{th} link and its inertia matrix respectively.

It is clear that the ZMP trajectory is directly related to the linear and angular accelerations of the robot's links. As a result, it is also related to the joint accelerations. If the joint accelerations are discontinuous, the ZMP trajectory will be as well discontinuous, which causes an imbalance as the ZMP trajectory might leave the polygone of support.

Therefore, generating smooth acceleration trajectories is crucial for the stability of humanoid robots. Moreover, smooth acceleration trajectories put less stress on the joints motors.

III. Problem Formulation

In order to have a continuous acceleration, we need to guarantee that the acceleration variation (jerk) is less than a threshold. The main idea is to introduce the jerk as the control parameter of the inverse kinematics problem instead of the joint velocity.

The relationship between the position, velocity, acceleration and the jerk can be expressed by the following equation:

$$\begin{bmatrix} q_t \\ \dot{q}_t \\ \ddot{q}_t \end{bmatrix} = \begin{bmatrix} I_n & T\,I_n & \frac{T^2}{2}\,I_n \\ 0 & I_n & T\,I_n \\ 0 & 0 & I_n \end{bmatrix} \begin{bmatrix} q_{t-1} \\ \dot{q}_{t-1} \\ \ddot{q}_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{T^3}{6} I_n \\ \frac{T^2}{2} I_n \\ T\,I_n \end{bmatrix} \dddot{q}_t \qquad (9)$$

Where $X_t = \begin{bmatrix} q_t & \dot{q}_t & \ddot{q}_t \end{bmatrix}^T$ is the vector that we need to compute, $X_{t-1} = \begin{bmatrix} q_{t-1} & \dot{q}_{t-1} & \ddot{q}_{t-1} \end{bmatrix}^T$ is the actual (known) configuration values. T is the sampling time (e.g. 5 ms for the humanoid robot HRP-2) and I_n is identity matrix of size n (number of degrees of freedom). The unknown parameter in Eq. (9) is the jerk vector $\dddot{q}_t$.

The objective is therefore to transform the inverse kinematic problem Eq. (2) into a function of $u_t = \dddot{q}_t$ instead of $\dot{q}_t$ and adding constraints on the jerk vector. To this end, we can use the following equation:

$$\dot{q}_t = \dot{q}_{t-1} + T\,\ddot{q}_{t-1} + \frac{T^2}{2} u_t \qquad (10)$$

The inverse kinematic problem becomes:

$$\begin{aligned} &\min_{u_t} u_t^T Q u_t \\ &\text{subject to} \\ &J u_t = \tilde{\dot{r}}_t \\ &A\,u_t \leq \tilde{b} \\ &u^- \leqslant u_t \leqslant u^+ \end{aligned} \qquad (11)$$

where:

$$\begin{aligned} \tilde{\dot{r}}_t &= \frac{2}{T^2}\left(\dot{r}_t - J\dot{q}_{t-1} - J\,T\,\ddot{q}_{t-1}\right) \\ \tilde{b} &= \frac{2}{T^2}\left(b - A\dot{q}_{t-1} - A\,T\,\ddot{q}_{t-1}\right) \end{aligned}$$

and u^-, u^+ are the lower and upper limits of the jerk vector.

IV. Implementation

The implementation algorithm can be summarized as follows:

1) Initial values: $X_0 = \begin{bmatrix} q_0 & 0 & 0 \end{bmatrix}^T$ and $t = T$.
2) Compute the jacobian matrix using the actual configuration (X_{t-T}).
3) Solve the optimization problem Eq. (11) and obtain $u_t = \dddot{q}_t$.
4) Compute the configuration vector X_t using Eq. (9)
5) t=t+T
6) Return to step 2

V. Experimental Results

In order to validate the proposed method, the boxing motion in [4] has been considered. The objectif is to follow the Cartesian trajectories of the robot's hands using the proposed algorithm in Section IV. These trajectories are obtained by the algorithm proposed in [4], the trajectoires of the head joints are kept without modification. Snapshots of the simulated motion is given in Fig. 1. An example of the obtained joint trajectories (left and right elbow trajectories) while considering the jerk limits (5000 $rad\,s^{-3}$) is presented in Fig. 2.

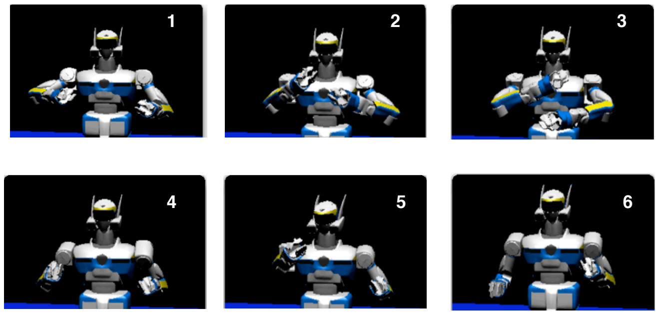

Fig. 1. Snapshots of the simulated motion.

VI. Conclusion and Future Work

In this paper, a new method for inverse kinematics while considering jerk limits has been proposed. This method guarantees that the trajectories of joints acceleration are smooth by defining a threshold for the jerk. A simulation result has been conducted and the efficiency of the algorithm has been pointed out. Futur work will focus on the realtime implementation of the proposed algorithm, the validation on walking patterns while avoiding self-collision [5], [6] and the validation on the real humanoid robot HRP-2.

References

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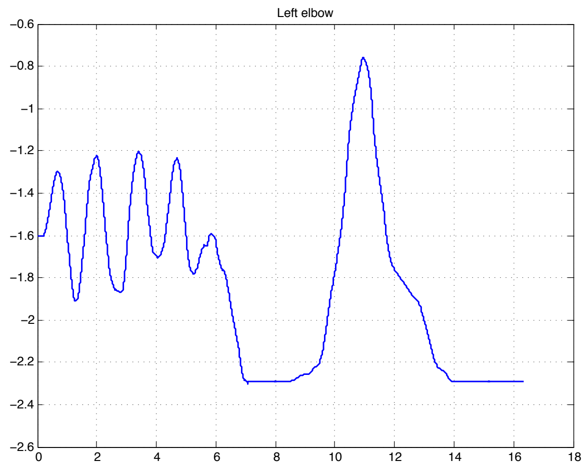
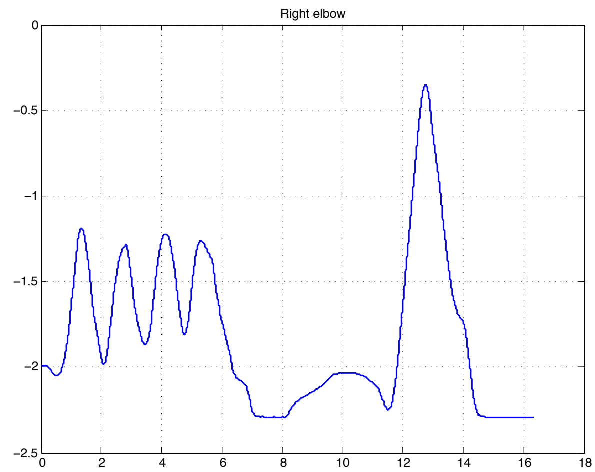
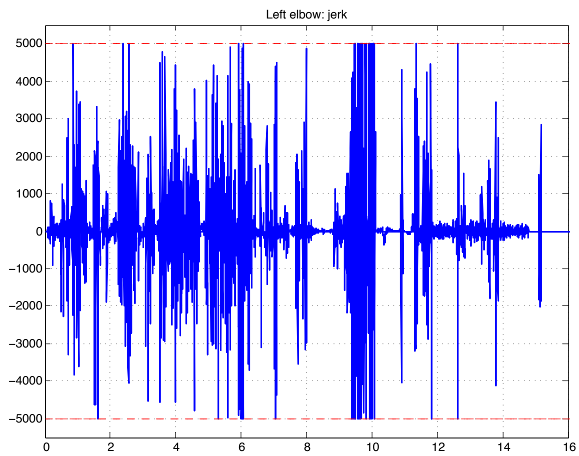


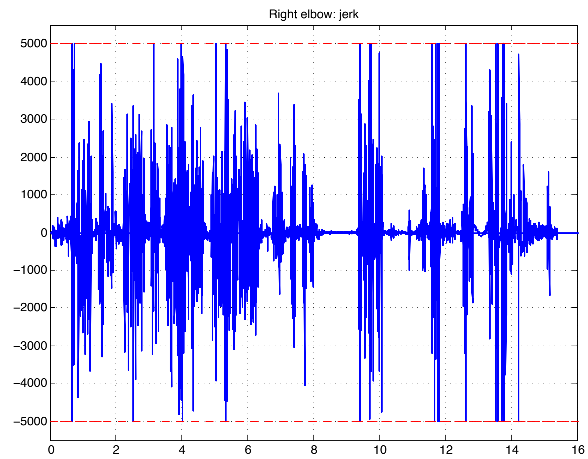
(a) Left elbow trajectory



(b) Right elbow trajectory



(c) Jerk of left elbow trajectory



(d) jerk of right elbow trajectory

Fig. 2. Right and left elbows trajectories.