

# Ground Reaction Forces Control for Torque-Controlled Quadruped Robots

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**Abstract**—Research into legged robotics is primarily motivated by the prospects of building machines that are able to navigate in challenging and complex environments that are predominantly non-flat. In this context, control of contact forces is fundamental to ensure stable contacts and stability of the robot. In this extended abstract we propose a planning/control framework for quasi-static walking for quadrupedal robots, implemented for a demanding application in which regulation of ground reaction forces is crucial for the success of the walking behaviour. Experimental results demonstrate that our 75-kg quadruped robot is able to walk inside two high-slope (50°) V-shaped walls; an achievement that to the authors’ best knowledge has never been presented before. The robot is able to distribute its weight among the stance legs while ensuring no foot slippage and fulfill the unilateral constraints of the contact forces. The approach makes use of a simplified model of the robot. Experimental evidence shows that even a lower dimensional model with the assumption of quasi-staticity is sufficient to perform the required task.

## I. INTRODUCTION

Current research on legged robots is motivated by their potential impact in real-world scenarios such as disaster recovery scenes. Such environments require systems capable of robustly negotiating uneven and sloped terrains. Despite remarkable advances in the theoretical tools [8, 7], to this date, experimental results have been limited to a few platforms and tasks, still not matching the complexity of the real world. Righetti et al. [11] experimented with walking up a slope of 26° with the *Little Dog* quadruped robot. On the quadruped robot StarLETH [3] Hutter et al. [5] used a contact-force optimization method to achieve static walking on a surface with approximately 40° inclination. Regarding contact force control in humanoid robots, so far research has mainly focused on balancing experiments on flat ground [6, 10, 13]. This substantial gap between simulation and reality is often due to the lack of high-fidelity joint torque control [5, 2, 1]. The contribution of this work is to combine different ideas from planning to control, and apply them to a challenging test case. Figure 1 presents the building blocks of our control framework. The *whole body controller* block is inspired by the work presented in [10, 3]. Its goal is to distribute the robot weight over the supporting contact points in an *optimal*

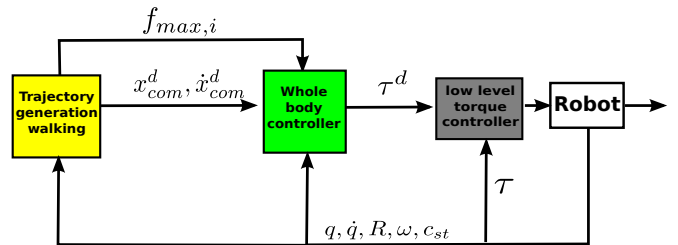


Fig. 1: Block diagram of our whole-body control framework. The motion generation block (yellow) computes the trajectories for CoM and joints to be input to the whole body controller (green) computes the reference torques for the low-level controller (grey).

manner. The approach is rather general as it can deal with any number of contacts as long as normal directions and friction coefficients are known or estimated. The method does not require contact force measurements and avoids joint torque discontinuities. The *motion generation* block computes desired trajectories for the center of mass (CoM), the base orientation and the swing foot to achieve a static walking pattern. Furthermore the robot is able to adapt to uneven surfaces, achieving a stable foothold and ensuring physical feasibility (e.g. not to violate the constraints of the stance feet). For the sake of brevity in this extended abstract we will present only the *whole body controller*.

## II. WHOLE BODY CONTROLLER WITH OPTIMIZATION OF GROUND REACTION FORCES

### A. Centroidal robot dynamics

Following the results presented in [9], the centroidal robot dynamics can be described as:

$$\dot{l} = m(\ddot{x}_{com} + g) = F_{com} \quad (1)$$

$$\dot{h} = I_G \dot{\omega}_G + \dot{I}_G \omega_G = \Gamma_{com} \quad (2)$$

where  $\dot{l}$  and  $\dot{h}$  are the rate of change of linear and angular momentum respectively,  $g \in \mathbb{R}^3$  is the gravity acceleration vector,  $m \in \mathbb{R}$  is the total robot mass,  $I_G \in \mathbb{R}^{3 \times 3}$  is the centroidal rotational inertia,  $\ddot{x}_{com} \in \mathbb{R}^3$  is the acceleration of

CoM,  $\dot{\omega}_G \in \mathbb{R}^3$  is the rotational acceleration of an equivalent rigid body with the inertia  $I_G$ , and finally  $F_{com} \in \mathbb{R}^3$  and  $\Gamma_{com} \in \mathbb{R}^3$  are the net external force and moment at the CoM, respectively. The design of the controller is based on the following assumptions.

- 1) we assume that  $I_G \dot{\omega}_G \simeq 0$ : this is reasonable because in our experiments the robot moves slowly.
- 2) since most of the robot's mass is located in its base (i.e. 47 out of 75 kg), we approximate the CoM  $x_{com}$  and the average angular velocity  $\omega_G$  of the whole robot with the CoM of the base  $x_{com-base}$ <sup>1</sup> and the angular velocity of the base  $\omega_b$ ,
- 3) since our platform has nearly point-like feet, we assume that it cannot generate moments at the contacts.
- 4) we assume that the ground reaction forces (GRFs) are the only external forces acting on the system.

Under these assumptions, expressing the net force and moment at the CoM as functions of the  $c$  GRFs (i.e.  $f_1, \dots, f_c \in \mathbb{R}^3$ , where  $c$  is the number of stance feet), we can rewrite (1) and (2) as:

$$m(\ddot{x}_{com} + g) = \sum_{i=1}^c f_i \quad (3)$$

$$I_G \dot{\omega}_b \simeq \sum_{i=1}^c (f_i \times p_{com,i}), \quad (4)$$

where  $p_{com,i} \in \mathbb{R}^3$  is a vector going from the CoM to the position of the  $i^{\text{th}}$  foot defined in an inertial world frame. These two equations describe how the GRFs affect the CoM acceleration and the angular acceleration of the robot's base.

### B. Control of CoM and base orientation

We compute the desired acceleration of the CoM  $\ddot{x}_{com}^d \in \mathbb{R}^3$  and the desired angular acceleration of the robot's base  $\dot{\omega}_b^d \in \mathbb{R}^3$  using a PD control law:

$$\ddot{x}_{com}^d = K_{pcom}(x_{com}^d - x_{com}) + K_{dcom}(\dot{x}_{com}^d - \dot{x}_{com}) \quad (5)$$

$$\dot{\omega}_b^d = K_{pbase}e(R_b^d R_b^\top) + K_{dbase}(\omega_b^d - \omega_b) \quad (6)$$

where  $x_{com}^d \in \mathbb{R}^3$  is the desired position of the CoM, and  $R_b \in \mathbb{R}^{3 \times 3}$  and  $R_b^d \in \mathbb{R}^{3 \times 3}$  are coordinate rotation matrices representing the actual and desired orientation of the base w.r.t. the world reference frame, respectively,  $e(\cdot) : \mathbb{R}^{3 \times 3} \rightarrow \mathbb{R}^3$  is a mapping from a rotation matrix to the associated rotation vector,  $\omega_b \in \mathbb{R}^3$  is the angular velocity of the base.

### C. Computation of desired GRFs

Given a desired value of the linear acceleration of the CoM and the angular acceleration of the robot's base, by rewriting (3) and (4) in matrix form we compute the desired GRFs by solving the following optimization problem, which is a quadratic program:

$$\begin{aligned} f^d = \underset{f \in \mathbb{R}^k}{\text{argmin}} & (Af - b)^\top S(Af - b) + \alpha f^\top W f \\ \text{s. t.} & \quad \underline{d} < Cf < \bar{d}, \end{aligned} \quad (7)$$

<sup>1</sup>In the following we keep using  $x_{com}$  even if in the implementation we actually used  $x_{com-base}$ .

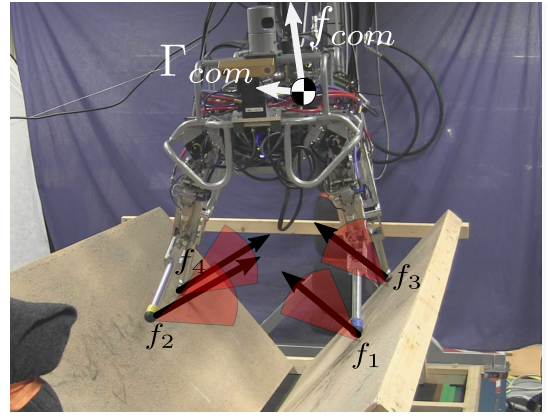


Fig. 2: HyQ walking inside a 50°-inclined groove. Desired wrench (force, moments) at the CoM is depicted in white. Ground reaction forces are in black, friction cone is light red.

where we exploit the redundancy of the solution to ensure the respect of the inequality constraints imposed by the friction cones and the unilaterality of the GRFs.  $W \in \mathbb{R}^{k \times k}$  is a weight matrix to keep the solution bounded and might be exploited also to optimize for torques while  $C \in \mathbb{R}^{p \times k}$  is the inequality constraint matrix. We approximate friction cones with square pyramids to express them with linear constraints. We compute the desired joint torques by mapping the desired GRFs  $f^d$  into joint space using the quasi-static assumption ( $\tau = -S J_c^\top f^d$ ).

## III. EXPERIMENTS

Extensive experiments have been carried out making the robot walk inside a 2.5m long V-shaped “horizontal groove” which is a good template to test the capability of our framework in controlling the ground reaction forces. The experimental platform used in this work is the HyQ quadruped robot [12] (Fig. 2). The robot weights 75 kg, has  $1m \times 0.5 \times 1m$  ( $L \times W \times H$ ) dimensions and is equipped with 12 actuated DoFs, i.e. 3 DoFs for each leg. All the joints of the robot are torque controlled with a high-performance low-level controller [1] which receives high level torque reference from the *whole body controller*. The base orientation control aims to maintain the robot's trunk horizontal during the walk. The control loop for the low-level torque controller ran at 800 Hz, whereas the whole-body controller ran at 133 Hz. We solved the optimization problem (7) in real-time using the open source software OOQP [4].

Fig. 3 plots the tracking of the contact forces of the left front foot. A feedback ratio on average  $< 18\%$ , demonstrates that our approach captures well the predominant robot dynamic behavior. Figure 4 shows the distribution of GRFs on all the legs for the same groove experiments. The GRFs are always inside the friction pyramid boundaries.

Note that the unilateral constraints on the contact forces implicitly restrict the CoP inside the convex hull of the support polygon.

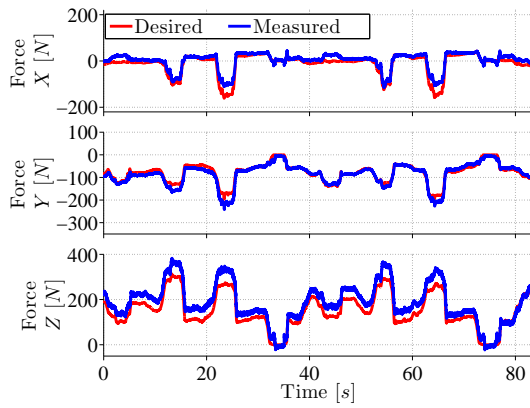


Fig. 3: Cartesian components of the contact forces in the left front leg. Red plots are the desired forces generated by the optimizer while blue plots are the actual contact forces.

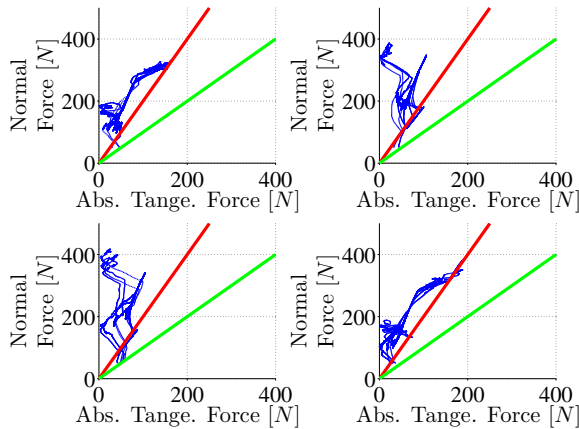


Fig. 4: Distribution of the contact forces at the four feet. The plots show the forces along the ground normal direction as a function of the norm of the tangential forces. The red lines represent the boundaries of a conservative friction cone set in the controller, which correspond to a friction coefficient  $\mu = 0.5$ , while the green lines represent the estimated one.

#### IV. CONCLUSIONS AND FUTURE WORK

We presented a control framework for quadrupedal quasi-static walking on high-sloped terrain, reporting experimental results on a torque-controlled quadrupedal robot. By direct control of the GRFs we were able to avoid slippage despite the high terrain inclination (i.e.  $50^\circ$ ). We believe that this capability is essential for the deployment of robots in adverse environments, such as mountains or disaster-recovery scenarios. Despite the simplifying assumptions the use of a lower dimensional model was sufficient to perform the task. The presented experiments show that the recent trend of force-based control frameworks can be used to perform locomotion on high-slope terrain.

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