The special Euclidean group (3) is commonly know in the robotics literature as homogeneous transformations. The Lie algebra of SE(3), denoted $se(3)$, is identified by a 4×4 skew symmetric matrix of the form:

$$\begin{bmatrix} 0 & -\omega_3 & \omega_2 & v_1 \\ \omega_3 & 0 & -\omega_1 & v_2 \\ -\omega_2 & \omega_1 & 0 & v_3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \hat{\xi} \tag{1}$$

The mapping from $se(3)$ to $SE(3)$ is performed by the exponential formula $H = e^{\hat{\xi}}$ and a closed-form solution exists through the Rodriguez formula. We refer to the matrix $\hat{\xi}$ as a *twist*. Similar to Murray, we define the $\vee$ (vee) operator to extract the six-dimensional *twist coordinates* which parametrize a twist,

$$\begin{bmatrix} 0 & -\omega_3 & \omega_2 & v_1 \\ \omega_3 & 0 & -\omega_1 & v_2 \\ -\omega_2 & \omega_1 & 0 & v_3 \\ 0 & 0 & 0 & 0 \end{bmatrix}^{\vee} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = \xi \tag{2}$$

The motion between consecutive frames can be represented by right multiplication of H with a motion matrix M.

The adjoint operator provides a convenient method for transforming a twist from one coordinate frame to another. Given $M \in SE(3)$, the adjoint transform is a 6×6 matrix which transforms twists from one coordinate frame to another.

$$M = \begin{bmatrix} R & \mathbf{t} \\ \mathbf{0_{1\times3}} & 1 \end{bmatrix} \tag{3}$$

$$Ad(M) = \begin{bmatrix} R & \hat{\mathbf{t}}R \\ \mathbf{0_{3\times3}} & R \end{bmatrix} \tag{4}$$

The adjoint operator is invertible, and is given by:

$$Ad^{-1}(M) = \begin{bmatrix} R^T & -R^T\hat{\mathbf{t}} \\ \mathbf{0_{3\times3}} & R^T \end{bmatrix} \tag{5}$$